International Journal of
Geohazards and Environment
http://ijge.camdemia.ca, ijge@camdemia.ca

IJGE 2015 **1**(**2**): 53-62

Available at http://ojs.library.dal.ca/ijge

A Phenomenological Model of Brittle Rocks under Uniaxial Compression

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Abstract: A quantitative model was recently proposed by Peng et al (2015) to characterize the crack closure behavior of rocks. The model can simulate the initial nonlinearity of stress–strain curves in uniaxial and triaxial compressions. However, the crack propagation behavior near peak and in the post-peak deformation stage cannot be captured by the model. This paper extends the model to simulate the complete stress–strain curves of rocks under uniaxial compression. A phenomenological damage model, which uses a logistic function to describe the damage evolution, is adopted to model the pre-peak and post-peak deformation stages beyond the crack closure stage. Combining the crack closure model and the damage model yields a new phenomenological model. A uniform continuity condition is used to ensure that the stress–strain curve is smooth and continuous at the junction point of the crack closure model and the damage model. The proposed model has four model parameters, which can be calibrated using laboratory test data. The uniaxial compression tests of the Carrara marble under different heating cycles are simulated to verify the proposed model. The simulated stress–strain curves agree well with the test data, from initial crack closure to near peak and post peak, suggesting that the model can be used for simulating the entire deformation stage of brittle rocks with different degrees of initial microcrack damage.

Keywords: crack closure behavior, microcrack damage, damage mechanics, phenomenological model, uniaxial compression test

1 Introduction

To mitigate and control geohazards such as slope failure, tunnel instability and rockburst, it is fundamental to study strength and deformation behavior of rocks. Estimation of rock strength and deformation parameters is necessary to develop constitutive models which are important for engineering design and analysis. An essential step in rock property characterization is to collect representative rock samples and conduct laboratory test on the specimens. It is found from numerous laboratory compression test results that the deformation behavior of intact rocks is mainly related to closure, generation and interaction of microcracks developed inside the rocks (Brace et al 1966, Bieniawski 1967, Lajtai and Lajtai 1974, Martin and Chandler 1994, Eberhardt et al 1998, Cai et al 2004, Diederichs et al 2004, Diederichs and Martin 2010). The progressive failure of brittle rocks can be divided into several stages

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DOI: 10.15273/ijge.2015.02.007

from crack closure to crack coalescence and several characteristic stress levels can be identified (see Fig. 1).

The crack closure behavior is an important part in the complete rock deformation process, which occurs at the initial loading stage. As shown in Fig. 1, the stress–strain curve of this part is usually non-linear (concave). In particular, when a specimen experiences a high degree of thermal loading, the initial stress–strain curve will be strongly non-linear (Rosengren and Jaeger 1968, Homand-Etienne and Houpert 1989, Mahmutoglu 1998, Du et al 2004, Keshavarz et al 2010). Because the nonlinearity is associated with

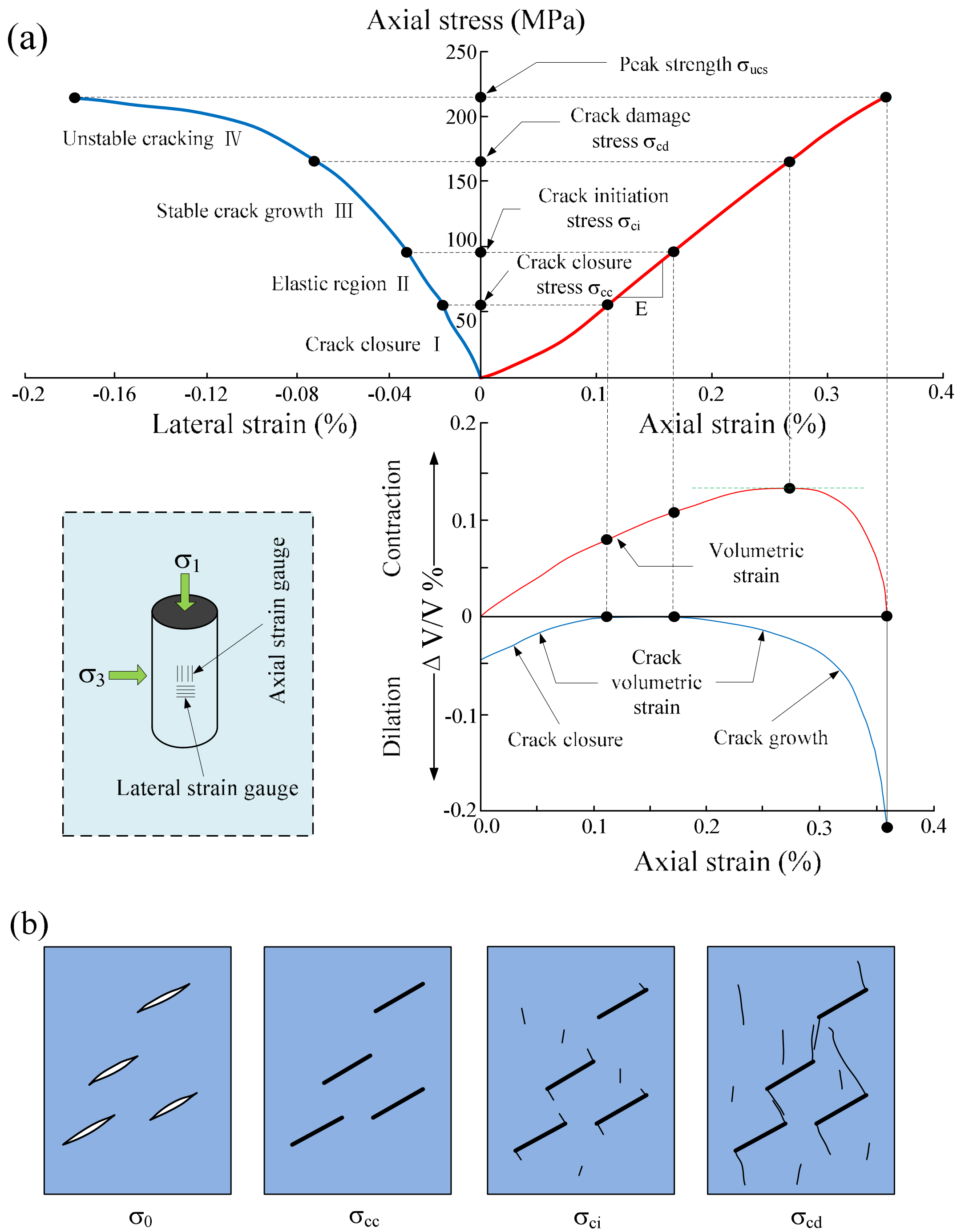

Fig. 1 A stress–strain diagram showing various stages of rock deformation in uniaxial compression tests (modified from Martin (1993) and Cai et al (2004)). (a) The regions of crack closure and growth, volumetric strain response, and the change of crack volumetric strain are illustrated; (b) a schematic representation showing different deformation behaviors including crack closure, crack initiation, crack propagation and coalescence

increased microcracking, the peak strength and the Young's modulus of the rock will be reduced. As such, the crack closure behavior can be used to evaluate the degree of initial microcrack damage in the rock and attention should be paid to this portion of the stress–strain curve in the development of a constitutive model.

Many researchers have studied the crack closure behavior of rocks. Corkum and Martin (2007) identified the nonlinear stress–strain behavior in the initial deformation stage of the Opalinus Clay and proposed a conceptual model that related the Young's modulus to stress. David et al (2012) proposed a sliding crack model for modeling nonlinearity and hysteresis in the stress–strain curves of uniaxial compression. The proposed model has four parameters including the elastic modulus of undamaged rock, the crack density, the characteristic aspect ratio, and the friction coefficient of crack surface. Recently, Peng et al (2015) studied the crack closure behavior of several rocks and proposed a quantitative model for characterizing the crack closure effect. The proposed crack closure model is feasible to model the initial nonlinearity of the stress–strain curve in uniaxial and triaxial compressions. However, the crack propagation behavior near peak and in the post-peak deformation stage cannot be captured by the model.

Continuum Damage Mechanics (CDM) based models are widely used to model deformation behaviors of brittle rocks without resorting to a microscopic description of the microcrack initiation and propagation process, which would be too complex for practical engineering application. A damage model uses a state variable to represent the effect of damage on the stiffness and strength of a rock (Krajcinovic and Fonseka 1981, Lemaitre 1985). The main issue in using a damage model lies in defining the damage variable and its evolution. Liu (2014) recently proposed a logistic model for the evolution function of the damage variable and established a phenomenological damage model. The model can capture well the damage evolution of rocks in the deformation process beyond the crack closure stage. In this paper, we improve and complete the crack closure model proposed by Peng et al (2015). The proposed crack closure model is combined with the phenomenological damage model (Liu 2014) to capture complete stress–strain curves of rocks from crack closure, elastic deformation, crack initiation and propagation, to the post-peak deformation stages. A method is developed to integrate these two models smoothly and continuously, resulting in a new phenomenological model that can capture the complete stress–strain curve of rocks under uniaxial compression.

2 Modeling of Crack Closure

Mahmutoglu (1998) studied the influence of heating cycles on the mechanical properties of the Carrara marble. In each heating cycle, a specimen was heated to 600 °C and then cooled down to room temperature (25 °C). Uniaxial compression tests were carried out for specimens that went through 0, 1, 2, 4, 8 and 16 heating cycles. In this section, the crack closure model proposed by Peng et al (2015) is reviewed and illustrated by interpreting uniaxial compression test data of the Carrara marble.

In the model by Peng et al (2015), the total axial strain at any point (see point A in Fig. 2(a)) in the stress–strain curve can be divided into two parts which are termed as the matrix axial strain ε_1^m and the crack axial strain ε_1^c, i.e.,

$$\varepsilon_1 = \varepsilon_1^m + \varepsilon_1^c \tag{1}$$

The matrix axial strain is the axial strain of the rock without microcracks, and it is linearly related to the elastic modulus of the rock by,

$$\varepsilon_1^m = \frac{\sigma_1}{E} \tag{2}$$

where σ_1 is the axial stress applied on the rock, and E is the Young's modulus of the rock which can be determined from the linear portion of the tested stress–strain curve.

A negative exponent function (Fig. 2(b)) is used in the model to represent the relationship between the crack axial strain and the applied axial stress, i.e.,

$$\varepsilon_1^c = V_m\left[1-\exp\left(-\frac{\sigma_1}{n}\right)\right] \tag{3}$$

where V_m is a model parameter that presents the maximum closure strain of microcracks, and n is a model parameter with the unit of stress which controls the curvature of the fitted curve.

Substituting Eqs. (2) and (3) into Eq. (1) yields a phenomenological model which can be used to simulate the crack closure behavior in the initial stage of the stress–strain curve. The model is expressed as:

$$\varepsilon_1 = \frac{\sigma_1}{E} + V_m\left[1 - \exp\left(-\frac{\sigma_1}{n}\right)\right] \tag{4}$$

The parameters V_m and n in the model can be calibrated by fitting the test data using Eq. (3).

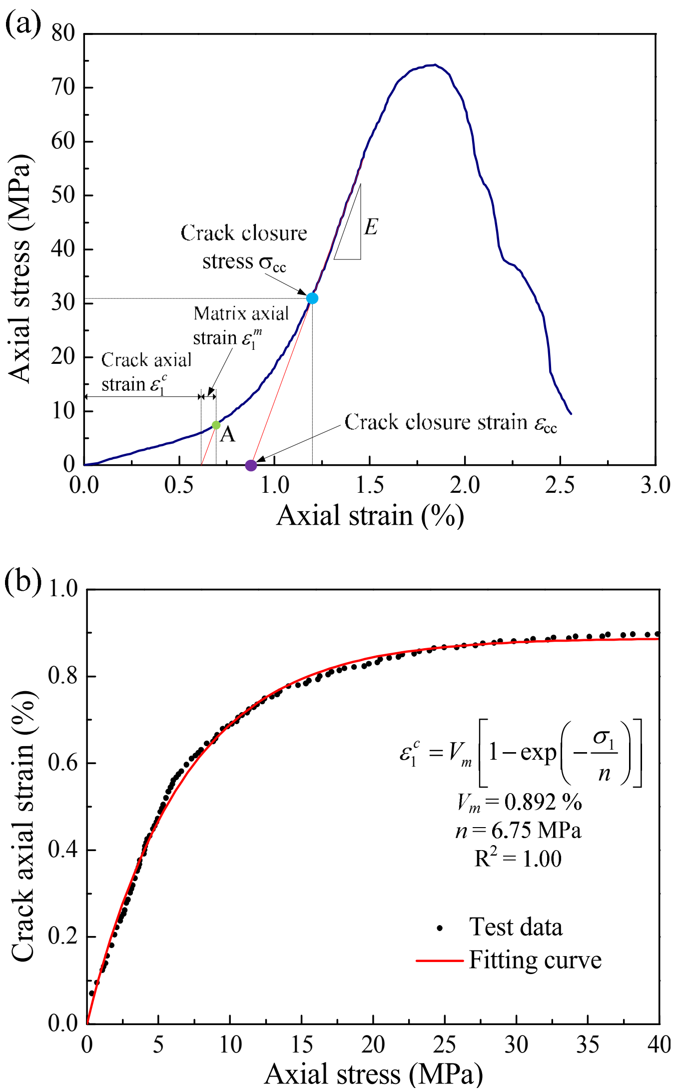

Fig. 2 Test data interpretation for the Carrara marble under one heating cycle. (a) Determination of the crack axial strain from the stress–strain curve. (b) Fitting the test data using a negative exponent function

Peng et al (2015) used the uniaxial compression test data of the Carrara marble to verify their proposed model and the simulated axial stress–axial strain curves are presented in Fig. 3. It is seen that when the stress magnitudes are not high compared with the peak strength, the simulated stress–strain curves agree well with the test data, indicating that the crack closure model can be used to model the crack closure behavior in uniaxial compression. However, the model given by Eq. (4) cannot simulate the stress–strain curve well near the peak strength. Furthermore, the post-peak behavior cannot be captured using the crack closure model. These problems are addressed by employing a phenomenological damage model which will be discussed in the next section.

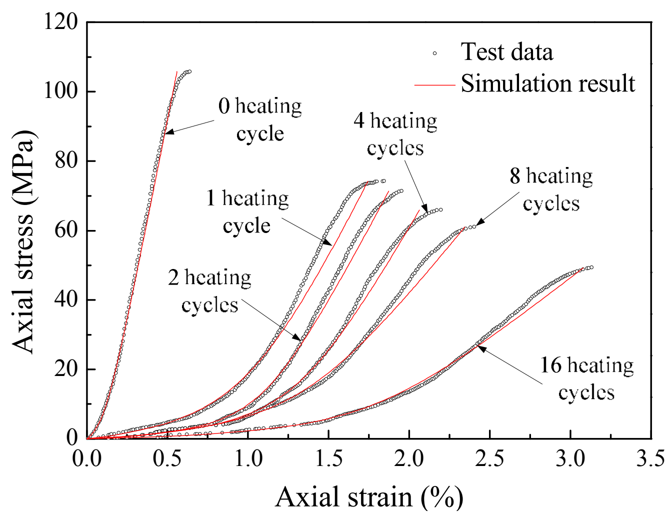

Fig. 3 Comparison of the simulated stress–strain curves with test data for the Carrara marble (after Peng et al (2015))

3 A Phenomenological Damage Model

3.1 Damage variable

In a CDM model, the change in Young's modulus is usually used to define damage variable D, i.e.,

$$D = 1 - \frac{E^*}{E} \tag{5}$$

where E^* is the apparent deformation modulus which represents the slope of any point in the stress–strain curve, and E is the Young's modulus of the rock. The Young's modulus E can be obtained from the linear elastic deformation portion of the stress–strain curve.

To make sure that the damage variable ranges from 0 to 1, a reference point is used in the determination of the apparent deformation modulus E^*. The crack closure strain ε_{cc} defined in Fig. 4 is used as the reference point in this paper. As shown in Fig. 4, the reference point is the intercept of the extension line of the linear portion of the stress–strain curve with the axial strain axis. The apparent deformation modulus E^* is then determined by the slope at any point in the stress–strain curve to the reference point. Take Point G in Fig. 4 for example, the apparent deformation modulus E^* at this point is expressed as:

$$E^* = \frac{\sigma_1}{\varepsilon_1 - \varepsilon_{cc}} \tag{6}$$

where σ_1 and ε_1 are respectively the axial stress and the axial strain of Point G in Fig. 4. One shortcoming of the method for determining the apparent deformation modulus E^* is that it is not applicable to the crack closure deformation stage.

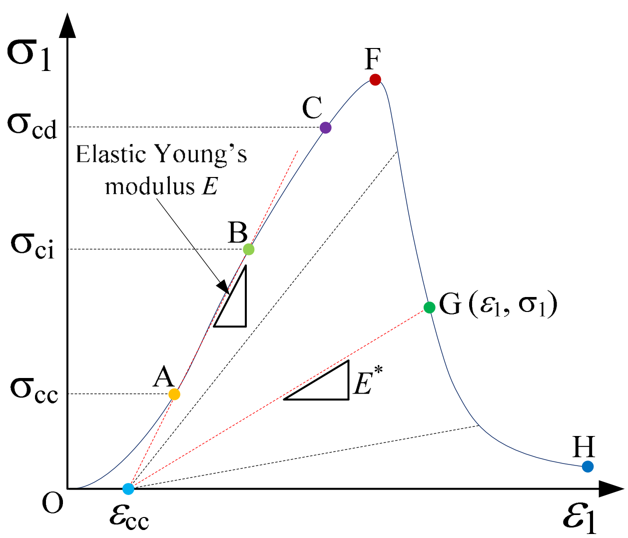

Fig. 4 A diagram showing the elastic Young's modulus and the apparent deformation modulus

Substituting Eq. (6) into Eq. (5) yields a damage model, i.e.,

$$\sigma_1 = (1 - D) E (\varepsilon_1 - \varepsilon_{cc}) \tag{7}$$

Determining the evolution function of the damage variable D is the main issue when using a damage model. In the following section, we discuss a logistic model for the evolution function of the damage variable.

3.2 A logistic model for damage evolution

Triaxial compression tests on Tennessee marble were carried out by Wawersik and Fairhurst (1970) to study the crack development in brittle rocks. Liu (2014) studied the Tennessee marble and other rocks and established a damage evolution function for the damage model. The test data of Tennessee marble in uniaxial compression is used in this section to illustrate the development of the damage evolution model. The apparent deformation modulus at any point beyond the crack closure stage in the stress–strain curve is determined by Eq. (6), and the variation of the normalized Young's modulus with the axial strain can be obtained (Fig. 5).

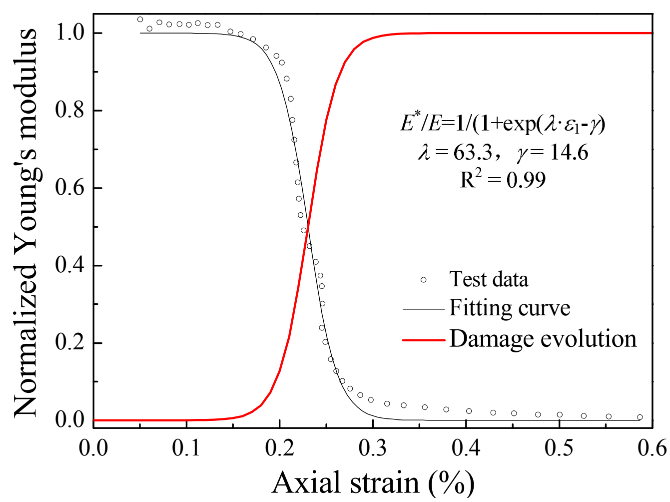

Fig. 5 Variation of the normalized Young's modulus with the axial strain. The damage evolution with the axial strain is also plotted

It is seen from Fig. 5 that the relationship between the normalized Young's modulus and the axial strain obeys a logistic function. Thus, a logistic model can be used to fit the relationship between the normalized Young's modulus and the axial strain. Fig. 5 also presents the damage evolution with the axial strain. The fitting result shows that the logistic model can capture the damage evolution of rocks under uniaxial compression.

Based on the above analysis, the normalized Young's modulus is expressed as

$$\frac{E^*}{E} = \frac{1}{1+e^{\lambda\varepsilon_1-\gamma}} \tag{8}$$

where λ and γ are fitting parameters of the logistic model.

A phenomenological damage model, which represents the mechanical response of rocks under uniaxial compression, is obtained by substituting Eqs. (5) and (8) into Eq. (7), which yields:

$$\sigma_1 = \frac{E\left(\varepsilon_1-\varepsilon_{cc}\right)}{1+e^{\lambda\varepsilon_1-\gamma}} \tag{9}$$

The above model is used to reproduce the uniaxial compression test result of the Tennessee marble (Fig. 6). It is seen that the simulated stress–strain curve agrees well with the test data, indicating that logistic-model-based phenomenological damage model can capture the uniaxial compression behavior of rocks. However, as mentioned above, the determination of the apparent deformation modulus E^* is only applicable to test data beyond the crack closure stage. In other words, the crack closure behavior cannot be captured using the phenomenological damage model. As seen in Fig. 6, the crack closure deformation stage in the initial stress–strain curve is represented as a straight line

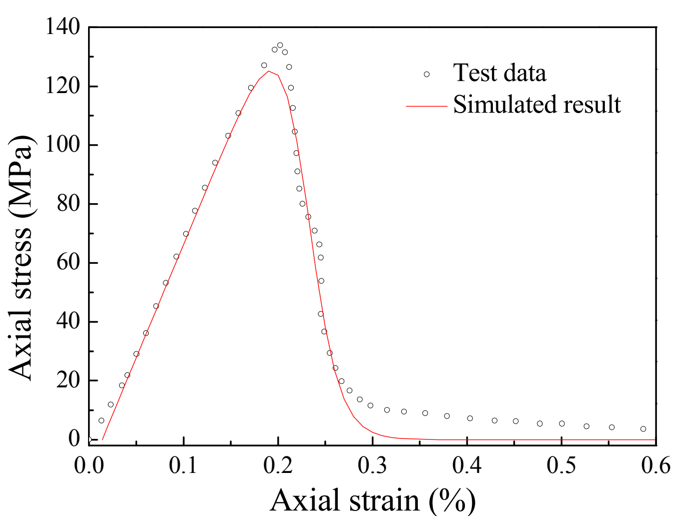

Fig. 6 Comparison of the tested and simulated stress–strain curves of the Tennessee marble

starting from the reference point (defined by the crack closure strain ε_{cc}) when the phenomenological damage model is used. If a rock, such as the thermal damaged Carrara marble, contains a large amount of microcracks, the nonlinearity in the initial deformation stage is strong. The phenomenological damage model cannot be used to simulate the complete stress–strain relations for this type of rock. In the next section, we propose to combine the negative exponential model and the logistic-model-based phenomenological damage model to model the complete stress–strain curves of rocks from crack closure to peak and post-peak deformation stages.

4 A New Phenomenological Model for Modeling Complete Stress–Strain Relations of Brittle Rocks

4.1 Model description

Based on the analysis in the above two sections, it is found that the crack closure model (Eq. (4)) can capture the crack closure behavior of rocks and the logistic-model-based phenomenological damage model (Eq. (9)) can simulate the rock deformation behavior beyond the crack closure stage. A new phenomenological model which can capture both the crack closure behavior during the initial deformation stage and the rest of the deformation stages of brittle rocks under uniaxial compression is proposed by combining these two models. The proposed model is expressed in the form of two segmented curves, i.e.,

$$\varepsilon_1 = \frac{\sigma_1}{E} + V_m\left(1-e^{-\sigma_1/n}\right) \quad \text{if } \varepsilon_1 < \varepsilon_0 \tag{10a}$$

$$\sigma_1 = \frac{E\left(\varepsilon_1-\varepsilon_{cc}\right)}{1+e^{\lambda\varepsilon_1-\gamma}} \quad \text{if } \varepsilon_1 > \varepsilon_0 \tag{10b}$$

where σ_1 and ε_1 are the axial stress and the axial strain during uniaxial compression of rocks, respectively, E is the Young's modulus, ε_{cc} is the crack closure strain, V_m, n, λ and γ are model parameters, which can be calibrated using test data, and ε_0 is the strain at the junction point of the two curves.

To ensure that the simulated stress–strain curve is smooth and continuous at the junction point ε_0, the uniform continuity condition should be satisfied. A translation method is used to satisfy the uniform continuity condition. The simulated crack closure curve by the crack closure model is kept unchanged, and the curve simulated by the logistic-model-based phenomenological damage model (which will be termed as the S curve in the following discussion) is translated by altering the crack closure strain value ε_{cc}. The point where the two curves meet and the tangents of the two curves are equal is the junction point ε_0. As shown in Fig. 7, the methodology is summarized as follows:

a. Determine the initial crack closure strain ε_{cc} based on the tested stress-strain curve, and fit the model parameters in the proposed model.

b. Translate the S curve and judge whether the crack closure curve is tangent to the S curve or not. The point where the two curves are tangent to each other is taken as the junction point ε_0. Otherwise, translate the S curve by adding a small value $\Delta\varepsilon_{cc}$ to the initial crack closure strain ε_{cc}.

c. Re-calculate the parameters in the damage model and update the simulated S curve.

d. Repeat Steps b and c until the junction point ε_0 is obtained.

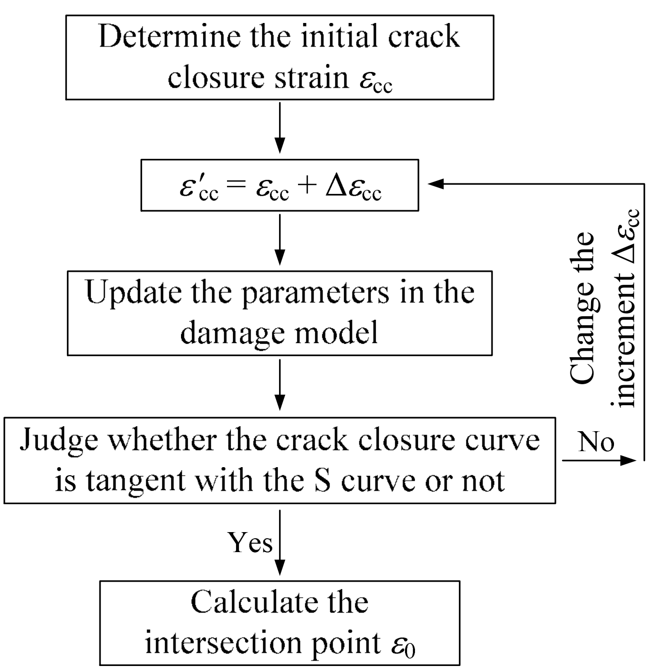

Fig. 7 A flow chart showing the determination of the junction point from the uniform continuity condition

The above procedure has been implemented into Matlab (Mathworks 2013) to automate the process. The uniform continuity condition can be fulfilled using the methodology presented above. The proposed phenomenological model is verified by simulating the uniaxial compression tests of the Carrara marble under different heating cycles in the next subsection.

4.2 Model verification

The proposed phenomenological model is used to simulate the stress–strain curves of the Carrara marble under different heating cycles, and the simulation results are presented in Fig. 8. The results show that the proposed model can represent the mechanical response of the Carrara marble from crack closure to peak and post-peak stages satisfactorily. Using the crack closure model alone, the initial crack closure behavior was well captured but near peak and post-peak behaviors could not be simulated. This problem has been solved by combining the crack closure model with the damage model. The parameters used in the model are presented in Table 1. It is seen that the updated crack closure strain ε'_{cc} is slightly smaller than the initial value ε_{cc}, indicating that a translation of the S curve to the left had occurred. Meanwhile, the fitted parameter V_m is very close to the initial crack closure strain value ε_{cc}. The crack closure strain, which is determined from the laboratory test results, can be used to estimate parameter V_m in the crack closure model.

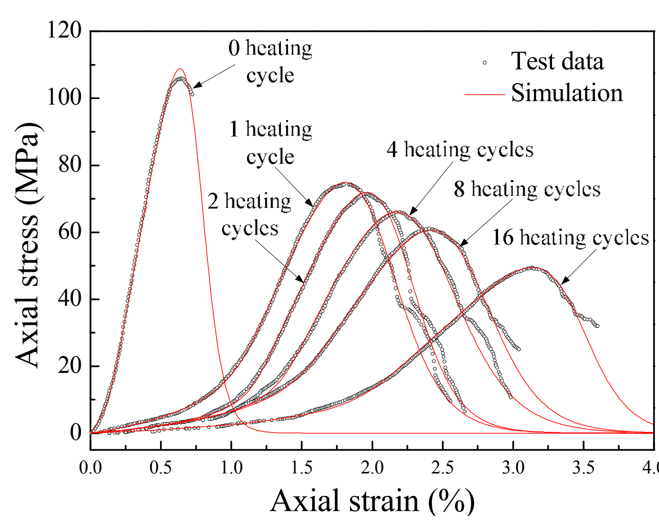

Fig. 8 Comparison of the simulated complete stress–strain curves with the test data of the Carrara marble

Table 1 Summary of the fitted parameters in the proposed phenomenological model

Heating cycles	UCS (MPa)	*E* (GPa)	Crack closure model			Damage model			ε_0 (%)	ε_{cc} (%)	ε'_{cc} (%)
			V_m (%)	*n* (MPa)	R^2	λ	γ	R^2			
0	105.8	23.7	0.104	6.58	0.98	13.70	10.53	0.94	0.302	0.104	0.103
1	74.2	10.0	0.892	6.75	1.00	5.74	11.84	0.99	1.310	0.897	0.885
2	71.3	9.1	1.020	3.79	0.98	6.45	14.27	1.00	1.356	1.033	1.019
4	65.9	8.6	1.201	4.48	0.98	4.71	11.52	1.00	1.599	1.222	1.194
8	61.0	6.2	1.252	5.12	0.98	5.26	14.38	0.99	1.791	1.260	1.248
16	49.3	3.8	1.682	3.23	0.99	6.52	22.58	0.98	2.420	1.708	1.681

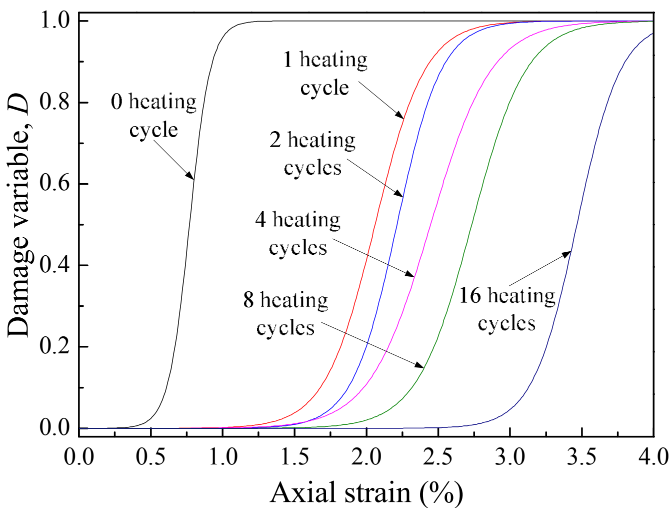

Fig. 9 Damage evolution with the axial strain under different heating cycles

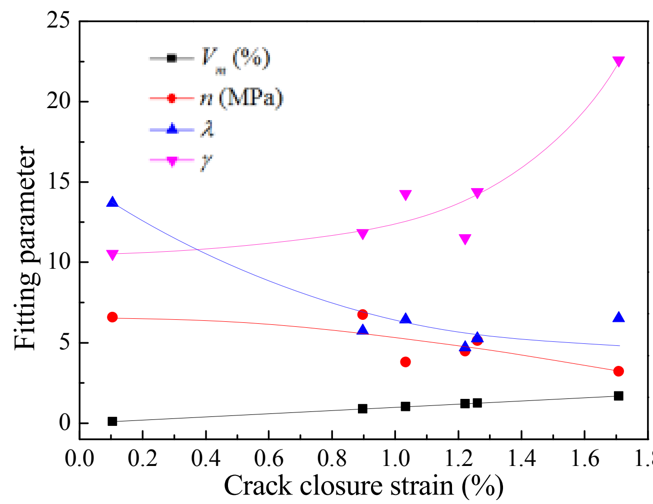

Fig. 10 Relationship between the fitting parameters and the crack closure strain

Figure 9 shows the evolution of the damage variable *D* (Eq. (5)) with the axial strain for the Carrara marble under different heating cycles. The damage evolutions basically follow a similar pattern for different heating cycles. A rock can be changed from a non-damage state to a damage state by thermal heating. More microcracks were introduced into the specimens with increasing heating cycles, leading to more axial strain.

The four fitting parameters in the proposed phenomenological model are correlated to the crack closure strain and the result is presented in Fig. 10. Both parameters V_m and γ show an increasing trend with the increase of the initial damage and the trend line for parameter V_m is almost linear. Parameters *n* and λ decrease with increasing initial damage.

5 Conclusions

The crack closure behavior is an important part of the complete rock deformation process. A crack closure model proposed by Peng et al (2015) can simulate the initial nonlinearity of the stress–strain curve in uniaxial and triaxial compressions. However, the crack propagation behavior near peak and in the post-peak deformation stages cannot be captured by the crack closure model alone. In order to simulate the complete stress–strain behavior of rocks under uniaxial compression, the crack closure model is extended by combining it with a phenomenological damage model. In the damage model, the degradation of the Young's modulus is defined as the damage variable, and a logistic model is used to characterize the damage evolution. A method is developed to satisfy the uniform continuity condition so as to ensure that the crack closure model and the damage model connect smoothly at the junction point. Because the microcrack-induced initial damage can be reflected in the crack closure stage, the new phenomenological model can be used to simulate the mechanical behavior of rocks with different initial damage levels. By simulating the uniaxial compression test results of the Carrara marble under different heating cycles, it is found that the new phenomenological model can capture the complete stress–strain behavior of brittle rocks from the crack closure to peak and post-peak deformation stages.

Acknowledgement

The research work presented in this paper is in part supported by the National Basic Research Program of China (''973'' Program, Grant No. 2011CB013501) and the Natural Sciences and Engineering Research Council (NSERC) of Canada. The authors are grateful for these financial supports. The first author is also thankful to the China Scholarship Council (CSC) for providing him a scholarship during his study in Canada.

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