

**Technical Notes** 

# Impact Force of Boulders Conveyed in Debris Flows on Bridge Piers and Collision Protection Measures

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Abstract: It is quite common in highway engineering that building a bridge across a debris flow gully to prevent roadbed from damage by strike of debris flows. As bridges are designed with the purpose to protect their piers against debris flows, it is crucially important for engineers to determine the magnitude of the Impact Force Exerted by Boulders Transported (IFEBT) in rush torrents. In view of the theory of energy conservation, a formula is introduced in this paper to calculate the IFEBT with appreciable improvement compared to the commonly used equations, in which only the two types of structures (cantilever and simply supported) are taken into account in modelling. The Thornton elastoplastic contact criterion is included in the formula in consideration of buffer effect of two-phase debris flow on bridge piers and dynamic responses of bridge upper-structure. Comparisons on calculation accuracy are elaborately made between our improved formula and previous methods in a case study of Den Jigou Bridge. It is found that according to our proposed method the values of IFEBT obtained in circumstances of varied velocity and boulders sizes are lower than the ones calculated by previous methods. Providing the depth of debris flow body in the two-phase condition is up to 2.4 cm, there is a considerable decrease of 21% in the value of IFEBT. In the meantime, a decrease of 1.4% in the IFEBT value is attained in consideration of the inertia force of the bridge's upper-structure. In addition, it is feasible to dissipate impact energy of IFEBT when low elasticity modulus and high decrement material are used in practical engineering.

Keywords: debris flow, impact force, boulders transport, bridge pier, energy dissipation

### 1 Introduction

There are numerous occurrences in mountainous areas of Southwestern China that bridges are ruined or seriously damaged at piers by boulders transported by debris flows. The debris flow occurred in Suo Tong gully along Sichuan-Tibet highway in 1991 is a typical example, during which bridge piers at the eastern side of the gully were completely destroyed, and the upper section of the bridge at a height of ten meters above the gully floor were broken (Pan et al 2009). Another tragic incident occurred on July 9, 1981, in Li Ziyida Gully in Guo Luo County, Sichuan Province, where sediments were discharged out of the gully mouth in a volume of  $8.4 \times 10^5$  m<sup>3</sup> and the bridge piers were cut utterly by tremendous boulders riding on the crest of waves of debris flows. It derailed a train currently traversing from Ge Lipin to Chengdu, causing more than four hundred causalities (Wang et al 2009).

The impact force induced by floating boulders trapped in debris flow hitting on bridge piers is a fundamental coefficient in engineering design. It has to be determined prior to the design of a bridge and layout in order to secure bridge piers and structure. There are four commonly used methods to calculate the Impact Force Exerted by Boulders Transported (IFEBT). The first method takes into account of the displacement of bridge structure (bridge is modelled as a whole) caused by IFEBT, where the structure is simplified as an ideal mechanical model in simulation (Cheng and Wang 1983, Wu et al 1997); The second method considers the striking velocity and the boulder sizes, and treats bridge structure simply as a supported beam or cantilever beam (Tang 1994, Hunger and Cantwell 1990). The third method assumes the surging boulders and the suffered bridge as perfectly elastic bodies, in which the reciprocal collision between boulders and the bridge has infinite contact radius of curvature, and their mass are also relatively large (Johnson 1985, Thornton and Ning 1997). The last one regards boulder and shocked object individually as rigid body and plastic body. It is only applicable to the old building to be impacted (He et al 2007, 2009a, Wang et al 2009).

There are deep deficiencies in the foresaid four methods. All of them have ignored the important phenomenon of IFEBT, and have excluded the buffer effect of two-phase debris flow during collision between boulders and bridge piers. Field investigation of Den Jigou debris flow has confirmed that fine particles filling between granular components act as a cushion buffer against large stone impacts on bridge piers. In this paper, a physical model is proposed to determine the IFEBT. This model considers buffer action of fine particles and dynamic response of upper structure of bridge. In order to surmount the disadvantages in previously used methods, an improved formula to calculate IFEBT was deduced based on the perspective of energy conservation by using the Thornton elastoplastic contact criterion. Compared to proposed method previous methods, this

reasonably avoids the shortcomings of excessive simplification as cantilever structures or simply supported structures. As an example, a Debris Flows Prevention Project in Den Jigou was studied, and IFEBT was carefully examined by testing the impact force of large stones acting on the Den Jigou Bridge piers under different parameters, followed by a comparison with results gained using previous methods. An optimal design of bridge piers and protective structures for the Den Jigou Bridge is put forward after the above studies.

### 2 Contact Mechanics Model based on the Thornton Hypothesis

Based on the Hertz contact theory, contact deformation comprises two components (Thornton and Ning 1997) (Fig. 1):

$$\delta = \delta_1 + \delta_2 \tag{1}$$

where  $\ddot{a}$  is normal compression,  $\ddot{a}_1 \ \ddot{a}_2$  are value of deformation of two contact objects, respectively.

$$a^2 = R\delta \tag{2}$$

where *a* is contact radius, *R* is equivalent radius,  $R_1$  and  $R_2$  are sphere radius of two contact objects. The following calculation equation expresses the relationship of  $R_1$  and  $R_2$  to *R*:

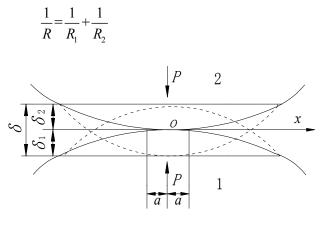


Fig. 1 The Hertz Contact Model

It is hypothesized that collision between a boulder and a bridge pier is satisfied with the Hertz contact condition. Assuming the contact meets the conditions of perfect elasticity, the relation between contact pressure and normal deformation can be expressed as:

$$P_{\rm e} = \frac{4E^*}{3} R^{\frac{1}{2}} \delta^{\frac{3}{2}}$$
(3)

where  $P_e$  is contact pressure,  $E^*$  is equivalent elastic modulus,  $E_1$  and  $E_2$  are elasticity modulus of two contact objects, and  $v_1$  and  $v_2$  are Poisson ratio of two contact objects. The following calculation equation expresses the relationship of  $E_1$  and  $E_2$  to  $E^*$ :

$$\frac{1}{E^*} = \frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2}$$

When the maximal contact stress is greater than the yield strength of materials, the plastic deformation zone would develop initially at the contact points with a relatively lower stress. The relation between initial yield stress and initial yield contact radius can be established by:

$$p_{\rm y} = \frac{2E^* a_{\rm y}}{\pi R} \tag{4}$$

where  $p_y$  is contact yield stress, and  $a_y$  is initial yield contact radius with regard to initial yield stress.

According to the Hertz theory, on the hypothesis that a material is satisfied with the Von Mises yield criterion, a calculation equation of initial yield stress is expressed as (Arattano and Franzi 2003, Braccesi and Landi 2006):

$$p_{v} = C_{v} Y \tag{5}$$

where  $C_v = 1.234 + 1.256v$ , v is Poisson ratio of material, and Y is yield stress of contact material.

The expression of elastoplastic normal pressure and normal compression proposed by Thornton can be defined as (Thornton 1998, Vuquoc and Lesburg 2001, He et al 2007):

$$P_{\rm ep} = P_{\rm y} + 2R\pi p_{\rm y} (\delta - \delta_{\rm y}) \tag{6}$$

where  $P_{ep}$  is normal contact stress,  $\delta_y$  is initial yield, and  $P_y$  is initial yield stress, i.e.  $P_y = \frac{4}{3} E^* R^{\frac{1}{2}} \delta_y^{\frac{3}{2}}$ .

#### **3** Model Concepts

Normally bridge piers are designed in a shape of circle or square for their cross section, and rooted beneath riverbed for sufficient bearing capacity. Therefore, in design simulation, bridge piers are usually treated as a simplified mechanical model, such as a structure of cantilever beams or simply supported beam. However, these simplifications do not reflect the actual circumstances, and seldom satisfy engineering needs for debris flow protection. It is necessary that inertia force incurred by debris flow should be included in the design simulation of the constraint effects of upper structures of bridges. Nevertheless, to determine the IFEBT, it must consider either the hitting effects of impact forces, or the buffer effects of collective fine particles in debris flow on a cantilever, or both. Providing the inertial forces are large enough, upper structure of bridge could be assumed as forces acting on a simply supported beam. Adversely if inertia forces are small, they are mechanically close to the effect of a cantilever beam.

In our physical model, boulders are mathematically simplified as a uniform sphere, which strike bridge piers with a velocity V at position X. The cross-sections of a bridge pier can be treated as a cantilever beam in a rectangular shape, or a specific cross-section in rectangular or circular at both ends of a beam.

In this research, it is assumed the crosssection of bridge piers is in a shape of rectangle. Fig. 2 demonstrates the schematic model calculating an impact force.

### 4 Calculation Model for Impact Force of Boulders in Debris Flow

### 4.1 The elastic deformation stage

When a boulder with a mass m is striking a pier with a velocity of V, flowing fine particle components of debris flow cause compressive deformation immediately on pier surface, and then elastic deformation ensues insider both the boulder and the pier, followed by bending deflection in bridge structure in the meantime. At this stage, the kinetic energy of a floating boulder can be divided into three parts: 1 the elastic

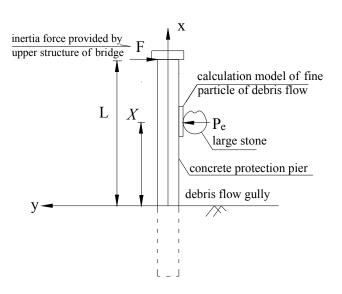


Fig. 2 The boulder impacting calculation model, where *L* is the pier height, *X* is the height of the impact force from reference coordinate, *F* is the inertial force of the upper structure of bridge, and  $\ddot{a}_0$  is the thickness of fine particles layer of bridge pier

energy from compression deformation generated by flowing fine particles in debris flow; ② the elastic energy of contact deformation; and ③ the deformation energy of reinforced concrete pier in bending. Under the action of inertia force provided by upper structure of bridge, the bending deformation energy of cantilever can be expressed as:

$$W_{\text{bend energy}} = P_{\text{e}} \cdot w(x) - F \cdot w_{\text{max}}$$
  
=  $\frac{P_{\text{e}}^2}{E_{\text{p}}I_{\text{p}}} (\frac{LX^2}{2} - \frac{X^3}{6}) - \frac{F^2 L^3}{3E_{\text{p}}I_{\text{p}}}$  (7)

where  $E_p I_p$  is the flexural rigidity of a concrete pier, X is the height of the impact position, F is the inertia force of a bridge upper structure, and  $P_e$  is the impact force of a large boulder.

According to the law of conservation of energy, Eq.(8) is presented as follows:

$$\frac{1}{2}mV^{2} = \int_{0}^{\delta_{0}} E_{\text{Fine-particles}} \delta d\delta + \int_{0}^{\delta_{1}} P_{e}(\delta) d\delta + \frac{P_{e}^{2}}{E_{p}I_{p}} (\frac{LX^{2}}{2} - \frac{X^{3}}{6}) - \frac{F^{2}L^{3}}{3E_{p}I_{p}}$$
(8)

where *m* is boulder mass, *V* is boulder impact velocity,  $\delta_1$  is elastic compression deformation of the Hertz contact,  $\delta_0$  is elastic compression deformation of fine particles,  $E_{\text{fine-particles}}$  is the elastic modulus of the fine particles.

Combination of the Eq.(3) and Eq.(8), Eq.(9) is presented as followed:

$$\frac{1}{2}mV^{2} = \frac{1}{2}E_{\text{fine-particles}}\delta_{0}^{2} + \frac{8E^{*}R^{\frac{1}{2}}}{15}\delta_{1}^{\frac{5}{2}} + \frac{16E^{*2}R\delta_{1}^{3}}{9E_{P}I_{P}}(\frac{LX^{2}}{2} - \frac{X^{3}}{6}) - \frac{F^{2}L^{3}}{3E_{P}I_{P}}$$
(9)

Impact speed can be calculated by the initial yield as:

$$V_{y} = \sqrt{\frac{\frac{2E_{\text{fine-particles}}\delta_{y}^{2}}{m} + \frac{16E^{*}}{15m}R^{\frac{1}{2}}\delta_{y}^{\frac{5}{2}}}{+\frac{32E^{*2}R}{9E_{p}I_{p}m}}} \left(\frac{LX^{2}}{2} - \frac{X^{3}}{6}\right) - \frac{2F^{2}L^{3}}{3mE_{p}I_{p}}}$$
(10)

#### 4.2 The elastic and plastic deformation stage

When  $V > V_y$ , plastic deformations take place at contact points. Through compression deformation, the fine matters between the gap of a large boulder and an protection pier will be further crushed after elastic deformation. After that, there is a collision between an protection pier and a boulder with plastic deformation.

Throughout the whole impact process, the kinetic energy of a boulder could be converted into four parts: (1) the elastic energy of fine particle matters at compression stage; (2) the elastic energy of contact deformation; (3) the plastic energy of contact deformation; and (4) bending deformation energy of the reinforced concrete pier (the impact of the process is shown in Fig. 3).

The following expression is obtained by combining the Eq.(3), Eq.(6) and Eq.(7) at the stage of structural elastic deformation with the law of conservation of energy:

$$\frac{1}{2}mV^{2} = \int_{0}^{\delta_{0}} E_{\text{fine-particles}} \delta d\delta + \int_{0}^{\delta_{1}} P_{e}(\delta) d\delta + \int_{0}^{\delta_{2}} P_{ep}(\delta) d\delta + \int_{0}^{\delta_{3}} P_{ep} \delta d\delta$$
(11)

where  $\delta_2$  is the deformation of the plastic compression, and  $\delta_3$  is the structure of bending deformation.

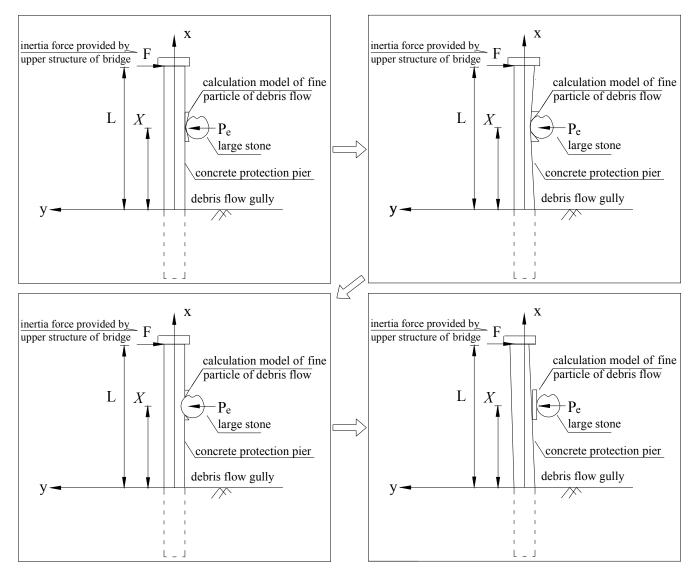


Fig. 3 Diagram of impact process course and elastoplastic deformation in piers

Further finishing,

$$\frac{1}{2}mV^{2} = \frac{1}{2}E_{\text{fine-particles}}\delta_{0}^{2} + \frac{8}{15}E^{*}R^{\frac{1}{2}}\delta_{1}^{\frac{5}{2}} + \left\{P_{y}(\delta_{2}-\delta_{1}) + \pi Rp_{y}(\delta_{2}-\delta_{1})^{2}\right\} + \frac{1}{2}[P_{y} + 2\pi Rp_{y}(\delta_{2}-\delta_{1})]\delta_{3}^{2}$$
(12)

In Eq.(12), the only unknown parameter  $\delta_2$  can be solved and the impact pressure  $P_{\text{max}}$  can be obtained by

$$P_{\max} = P_y + 2R\pi p_y (\delta_2 - \delta_1) \tag{13}$$

The Eq.(7)  $\sim$  Eq.(13) determine the IFEBT acting on bridge pier with the consideration of combining the buffer effect of fine particles in

debris flow body and the action of inertia force of super structure of a pier. If the equation is used to determine the impact force for a large boulder acting on an protection pier, the buffer action of fine particles cannot be ignored. In this case, the value of inertial force F shown in equations (8) and (9) is zero.

#### **5** Engineering Case and Discussions

#### 5.1 The Den Jigou bridge pier and debris flow

The Den Jigou debris flow gully is located in southern Wenchuan county, Sichuan province, China, 12.1 km away from the town. The flow direction of the gully is from north to west. The main channel length is 10.7 km with the upper reach in "V" shape and downstream "U" shape. The catchment area is 46.3 km<sup>2</sup>. The channel longitudinal gradient is 167‰. According to field sampling, particle sizes in debris deposition correspond to 50% solids content in the particle analysis chart, i.e.  $\delta_0$  value is 0.004 m. Field observation showed that the debris flows

discharge at frequencies of 1% and 2% was 780  $m^3/s$  and 652  $m^3/s$ , respectively. Debris flow flowing velocity is 4.59 m/s (Wang et al 2010).

The Den Jigou Bridge was built over the debris flow gully. Parameters of Den Jigou bridge piers and a large stone are provided in Table 1.

Bridge Pier				Large Stone					
$E_1$ (GPa)	I (m <sup>4</sup> )	L (m)	$\begin{array}{c} R_1 \\ (m) \end{array}$	$\rho$ (kg/m <sup>3</sup> )	V (m/s)	<i>R</i> <sub>2</sub> (m)	E <sub>2</sub> (GPa)	X (m)	v
22	0.785	12	1	2800	4.59	0.9	35	6	0.3

Table 1 Parameters of Den Jigou bridge piers and a large stone

It is assumed that F value, the inertia force of the upper structure of the Den Jigou bridge pier is 3,000 KN that is equal to weight of the bridge acting on pier multiplied by friction coefficient of bearing. If we did not consider the buffer effect of fine particles in debris flow and F value, the impact force of a large boulder acting on bridge pier is calculated to be 3.590 MN by Eq.(13). Using our theory with a consideration of the two factors, the impact force of a large boulder acting on bridge pier is calculated to be 3.448 MN, and the normal compression deformation to be 47.95 mm. The value of impact force obtained using our methods is reduced by 4.2%. Figs. 4 and 5 present the comparisons of the impact force based on our suggested method with the one based on the theory (Wang et al 2009), as opposed to varying velocities and radius.

From Figs. 4 and 5, it can be found that when considering the buffer action of fine particles in debris flow and inertia force of upper structure of bridge pier, the calculated impact force is lowered by  $2.3\% \sim 18\%$  than that without considering these two factors. The calculated results show that the buffer action of fine particles and upper structure inertia force of bridge pier have more influence on the impact force, and the impact force response is more noticeable by increasing radius than by changing speed velocity of a large stone. A study carried by He et al (2007) showed that the calculated impact force considering elastoplasticity of material was only 30% of elastic collision's. However, impact force

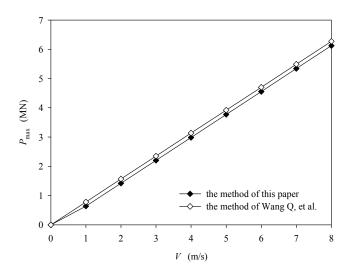


Fig. 4 Relationship between impact velocity V of a large boulder and  $P_{\text{max}}$ 

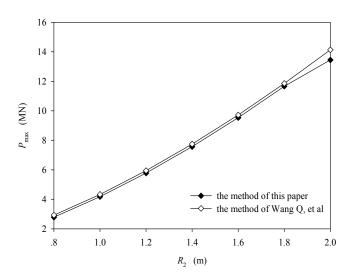


Fig. 5 Relationship between the radiuses  $R_2$  of boulders and  $P_{\text{max}}$ 

observed in engineering is less than 10% of elastic collision's (Zhang and Hunger 1996). The above-mentioned showed cases that the calculated impact force with solely considering the elastoplasticity of the material is larger than the observed impact force. In this study, the elastoplasticity of material as well as buffer action of fine particles in debris flow and inertia force of upper structure of bridge pier were considered in our calculated model, which generated lower values than those of other researches (Wang et al 2009, He et al 2009b).

We used our theoretical model to predict the effect of fine particles thickness and upper structure inertia force on large stone impact force (Figs. 6 and 7). It can be seen that the thickness of fine particles in debris flow has a greater effect on

piers than the impact force of boulders has. However, when inertia force of the bridge superstructure is included in calculation, the maximum impact force is reduced only by 1.4%. It is shown that the thickness of fine particles in debris flow has a large dissipative action on the impact energy of large stones in debris flow. This action should not be ignored when calculating. Compared with the influence factors of boulder hitting piers, the inertia force of upper structure of bridge piers does not have a greater influence than the debris flow fine particles.

Figs. 8 and 9 show large stone bump position and elastic modulus of bridge pier with respect to impact force of a large stone. Figs. 8 and 9 indicate that as the height of impact position increases, the impact force is reduced. However,

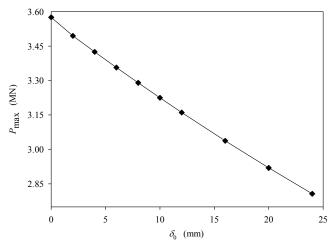


Fig. 6 Relationship of fine particles thickness $\delta_0$  in debris flow with  $P_{\text{max}}$ 

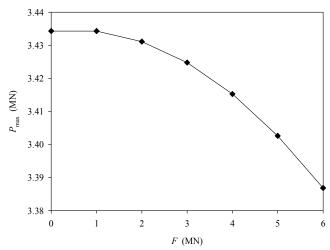


Fig. 7 Relationship of inertia force F of upper structure with  $P_{\text{max}}$ 

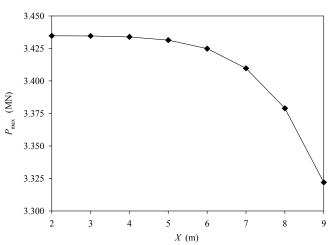


Fig. 8 Relationship of boulder bump position with impact force

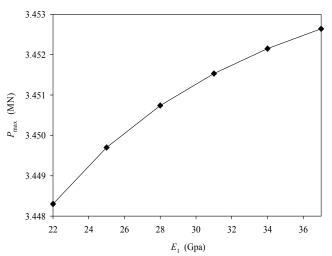


Fig. 9 Relationship of elastic modulus of bridge pier with impact force

the impact force is obviously not reduced within 6 m range. As the elastic modulus increases, the impact force increases. This shows that selection of the low elastic modulus and high compression materials for bridge pier protection is conducive to impact energy dissipation.

### 5.2 Den Jigou bridge pier protection

Based on above calculation, if a boulder with an equivalent radius of 0.9 m surging on the crest of debris flow hits directly at a bridge pier, it will produce a deformation of up to 47.95 mm and the pier may be broken. In order to ensure bridge safety, anti-collision pier structure was built in front of the bridge pier to resist boulder strike. The protection pier was designed with a height of 10 m, with 6 m beneath the ground (Figs. 10 and 11). Its front crash face is 2 m, and the back-end

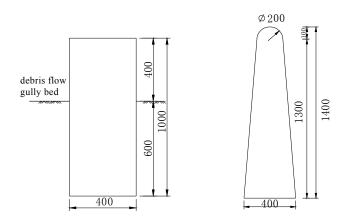


Fig. 10 Structure of an anti-collision pier (cm)



Fig. 11 The anti-collision pier applied for Den Jigou bridge pier protection

size is 4 m. There are three major advantages of this design towards protecting bridge pier. Firstly, it can avoid the direct collision between lower part of pier and boulders. Secondly, this shape can reduce contact area between protection pier and boulders. Lastly, the shape can change the movement direction of flowing boulder.

Assuming that the anti-collision pier and the bridge pier are built with same material, and the front face of anti-collision pier is 1 m high, the impact force incurred by a single boulder hitting on an anti-collision pier is 3.599 MN by equation (13).

Since the designed contact radius of the front face of anti-collision pier is 2 m, this size is more than safe. Based on above analysis, it is possible to decrease elastic modulus of pier concrete by using high compression materials. It will reduce the size of the anti-collision pier, thus resulting in material saving and cost reduction.

### 6 Conclusions

In this paper, a mechanical model for the calculation of the Impact Force Exerted by Transported Boulders (IFEBT) in debris flow was established. This is based on a combination which couples the effect of buffer action of fine particles in debris flow with upper structure dynamic response properties of bridge. In connection with the perspective of energy conservation, the combination approach turns out an equation to determine IFEBT by using the Thornton elastoplastic contact criterion. Compared to the previous methods, it avoids the load-carrying mode which is over-simplified for cantilever structures or simply supported structures. A case study was carried out for the Den Jigou gully to examine the feasibility of the improved equation in reality. Scenarios with varied parameters were introduced into the equation for verification of the calculated value of IFEBT in line with the previous results.

Some conclusions can be summarized as follows:

1) By considering the positive effect of buffer action of fine particles in debris flow on piers and the inertia force of the upper structure of bridge piers into our improved model, the value of impact force of debris flow on bridge is lower as compared with ones obtained without considering the above two factors. This result confirmed that the fine particles layer existing in debris flow as well as inertia force of the upper structure of bridge played a significant role in energy absorption during collisions.

2) The thickness of the fine particles layer in debris flow affected the value of IFEBT appreciably. Providing the thickness is 2.4 cm, the value of IFEBT calculated is reduced by 21%. Therefore, the existence of fine particles in debris flow cannot be ignored during the design of bridge protection against debris flow. When the bridge superstructure inertia force is taken into account in the mechanical model, the maximum impact force is reduced by 1.4%, which indicates that the inertia force of upper structure of bridge piers has less influence on IFEBT than the one by fine particles layer in debris flow.

3) As the height of contact face of IFEBT on piers increased, the impact force is correspondingly reduced. However, the impact force does not obviously reduce in the range of  $0 \sim 6$  m height of the piers. As the elastic modulus of the bridge pier increases, the impact force increases.

4) The anti-collision pier is an effective protective structure for bridge piers. If materials with low elastic modulus and high deformability are included in design of the anti-collision pier front, the reinforced concrete dimensions of anticollision pier can be reduced, resulting in materials saving and cost reduction.

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