Ὁ Λόγος Δέων: The Binding Ratio

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One of the most important foundations of the ontological principles detailed in Aristotle's *Metaphysics* may be found in his comparison of Plato's philosophy with that of the Pythagoreans. Aristotle finds Plato to be in agreement with the Pythagoreans with respect to the principle of numbers as the cause of existence, as he explains in the *Metaphysics*,

> "So therefore he called these kinds of beings Forms (ἰδέας), and he said that these are further things, and that the sensibles (αἰσθητῶν) are apart from these and are all spoken according to these. For by participation (μέθεξιν) many of these names are given to the Forms (εἴδεσι). And he changed only the names of participation (μέθεξιν). For the Pythagoreans said that beings exist by means of imitation of numbers (ἀριθμῶν), and Plato said that they did so by participation (μέθεξιν), changing the name."[1]

Aristotle indicates here that Plato emulates the Pythagoreans in the identification of numbers as primary causes, and that the apparent evolution in Plato's thought is little more than a change in terminology for a system that is otherwise functionally identical to its precursor. This characteristic is in strong opposition to Plato's objective, for he aims to allow for the existence of numbers independent of the sensible, and introduces the concepts of the Forms and of the Dyad to the λόγος of the unifying principle of the One. According to Plato, the Forms are abstract structures which exist prior to matter and are the causes of sensible things. The Dyad is the principle of infinity, otherness, and relation, a concept represented in the comparison between various objects with respect to characteristics such as distance and proportion. It is by this principle that objects may, through relation to each other, retain their distinction while simultaneously being united in the One. These concepts imply a transition from a purely mathematical paradigm of existence toward a system by which there exist causes which are prior even to number. These two strands of thought are not compatible, however, and we shall soon observe, the admixture of ἀριθμός and εἶδος as primary causes results in a number of problems, or more precisely, a problem of numbers.

The articulation of this system, however, is impeded by significant problems with Plato's understanding of the Forms and the Dyad in relation to numbers as a first cause. Aristotle cites the concepts expressed in Plato's *Phaedo*,[2] a dialogue in which Socrates,

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[1] Ar. Met. 987b. 7-13. All translations are by Ross, supplemented by other interpreters where stated.

[2] Ar. Met. 991b. 3-5.

through mathematical dialectic, convinces the Pythagorean pupils of Philolaus[3] of the survival of the soul following the death of the corporeal vessel. He refutes, for instance, the notion that the soul is a harmony, arguing thus,

> "Simmias, as I predict, believes and fears that lest the soul (ψυχὴ), [though it is more divine and more beautiful than the body], is destroyed first, [being a kind of harmony] (ἐν ἁρμονίας εἴδει): Cebes seems to agree with me on this, that the soul survives the body, but is [unknown to all whether] the soul (ψυχὴ), [often wears out] many bodies, will not be obliterated when the last body is destroyed, and that death itself is not the destruction of the soul (ψυχῆς ὄλεθρος), so it will never be destroyed by the body being destroyed."[4]

According to this passage, Simmias ascribes to a notion that the soul, while exhausting each body that it inhabits, is itself diminished until nothing remains of it. This position has the appearance of imposing a numerical limit upon the soul. Socrates, however, dismisses the claim that the soul may wane in such a manner, and thereby asserts the existence of substance beyond the restrictions of number. At an earlier position in the dialogue, however, Plato adds a possible weakness in his position, for Socrates, perhaps in his effort to express his argument in terms familiar to the Pythagoreans, speaks of the Forms in conjunction with mathematical relations. In his example he posits,

> "What then? Might one say that equal (ἴσα) things are unequal (ἄνισά), and that equality (ἰσότης) is inequality (ἀνισότης)? Certainly not, Socrates. It would seem then, that equal (ἴσα) things are not identical to the equal (ἴσον) itself. But from these things, with equalities (ἴσων) being different from the equal (ἴσου) itself, do you think,

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[3] Plat. Phaedo. 61d.

[4] Plat. Phaedo. 91c-d. Text enclosed by square parentheses has been amended with the assistance of Grube, 1997.
Socrates may here be understood as correcting a Neo-Pythagorean misinterpretation of Pythagorean doctrine, whereby the soul was believed to be diminished by reincarnation rather than perfected by it. In his work entitled *Philolaus of Croton: Pythagorean and Presocratic*, Carl A. Huffman proposes that Philolaus, whom the *Phaedo* recognizes as the mentor of Simmias and Cebes, described a soul, or ψυχή which was not immortal, but rather was contained within the heart merely as an "attunement" of the vital functions of the body in which it was held, with these functions being described as "limiters and unlimited." (Huffman, 1993, 229-32) Huffman states also that based on the fact that according to the thirteenth fragment of Philolaus, the "intelligence" is separated from ψυχή, and that there is also an immortal soul, possibly called δαίμων, however, he does not perceive any conclusive evidence that Philolaus held such a belief. Within the discourse of the *Phaedo*, there is evidence that Simmias and Cebes, who are Pythagoreans and pupils of Philolaus, believe in a type of ψυχή which is not entirely immortal, yet is not entirely bound to its initial body either. While it may reincarnate in another body after the death of a previous body, it exhausts each body, and with each successive incarnation it is expended further until it fades entirely from existence.

> as it would seem, that these are known to us? You speak most truthfully. Then surely it will either be like unto these things, or dissimilar to them, will it not? Undoubtedly."[5]

In this passage, Socrates indicates that prior to specific instances of equality and inequality, including those occurring in a mathematical context, there exist principles of the equal and unequal which are the causes of such particular comparisons. This passage is therefore a crucial component of Aristotle's criticism, for it suggests that according to Plato's doctrine, the abstract principles of the equal and the unequal may be most accurately identified as Forms.

Assuming that there is a Form for the equal and another for the unequal, these ought to exist prior to all other mathematical rules and operations. Without these ideas of mathematical identity, it would be impossible to affirm or deny any value with respect to any number, equation, or variable. There must also be intelligible principles for all simple mathematical operations[6] for otherwise, it would be impossible to establish consistent mathematical rules and formulae, and we would be unable to possess true knowledge of mathematics. One might suggest that there need not be distinct Forms for other types of mathematical principles, and that the concepts associated with the basic arithmetic operations might be contained within the Forms of the equal and the unequal. The absurdity of this position, however, is clear from the fact that it would imply that the Forms of the equal and unequal are composed of the ideas of the mathematical operations. If the Forms of the equal and unequal were composed as such, then the mathematical operations would be prior to the equal and unequal instead of being subsequent to them, and we have determined already that this result is problematic. It then follows that since there must be intelligible principles for specific mathematical operations, and these ideas cannot be contained within the Forms of the equal and unequal, there must be distinct Forms for the mathematical operations. It might still be argued that there need only be one Form in which all simple mathematical operations participate, but from this assertion, it would follow that the operations are merely specific things rather than intelligible principles. Even if a single Form existed for all simple operations, each operation of that kind would also participate in a distinct Form, which in turn participates in the Form of Simple Operations.

We have thus determined that from the existence of Forms for each of the equal and unequal, it follows that there will be a distinct Form for each of the simple mathematical operations. Thus, presupposing the existence of the Forms of equality and inequality, it must then be the case that there is a certain Form predicated of the relation of

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[5] Plat. Phaedo. 74c. (Interpreted with assistance from the translation according to G.M.A. Grube, in Cooper, 1997, 65, which reads, "But what of the equals themselves? Have they ever appeared unequal to you, or Equality to be Inequality? Never, Socrates. These equal things and the Equal itself are therefore not the same? I do not think they are the same at all, Socrates. But it is definitely from the equal things, though they are different from that Equal, that you have derived and grasped the knowledge of equality? Very true, Socrates. Whether it be like them or unlike them? Certainly."

[6] These operations may be called "simple" insofar as they are foundational operations and are therefore not composed of any other operations.

ratio. This Form of Ratio is central to the problems in Plato's doctrine of the Forms. Aristotle reveals that this problem is a function of the identification of the Forms as ratios, for in discussing the nature of the Forms, he states,

> "But if, indeed, the Forms (εἴδη) are numbers (ἀριθμοὶ) how will they be causes (αἴτιοι)? Are either of two different numbers beings, such that one such number is man, and one is Socrates, and one is Callias? Are these things then the cause of these men? For whether they are eternal or not is of no consequence. And if they are ratios of numbers in this state, as such is the case with harmony (συμφωνία), so that the sensible (δῆλον) is, at any rate, one of the things that are ratios (λόγοι). And if this, matter (ὕλη), is something manifest, then the numbers (ἀριθμοὶ) themselves will be ratios (λόγοι) of one thing to another."[7]

In this passage, Aristotle makes certain observations regarding the role of numbers in the doctrine of the Forms. He notes firstly that the Forms are considered numbers, and he states also that numbers will be ratios of something else. Thus, if Forms are numbers and numbers are ratios, it must be the case that the Forms are ratios. It has been concluded as well that based on the Forms of mathematical equality and inequality, there should be Forms for all types of mathematical relations, and these Forms will also be ratios. According to all these premises it follows that the Form of mathematical ratios is in fact a ratio itself. It seems, furthermore, as Aristotle explains, to be the case that in the doctrine of the Forms, sensible things are ratios of two numbers, while the Forms of which they partake are ratios of other numbers. Aristotle's explanation also suggests that according to the teachings of Plato, numbers are themselves ratios of one object to another.

Based upon this conclusion, we are presented with a mathematical ἀπορία beyond the problem stated by Aristotle, for it appears to be the case that mathematical ratios have certain numbers that are proper to them specifically. It suggests also that the ratio would have such a value apart from the quotient, as the operation itself may be understood to be a ratio of two numbers other than its operands. It would also be the case that the Form of Ratio, of which each specific ratio partakes, would have its own numerical value. It has also been suggested, however, that all of the Forms are ratios, that is to say, that each Form is the ratio of specific numerical operands. Beyond the numerical values of which the Form is a ratio, it will have its own specific value by means of the fact that it is a ratio of two numbers, and the Form of which it partakes, the Form of Ratio, will have a different value as well. It appears that the only way to explain this phenomenon is to say that the Forms are not all of the same rank, and that the Form of Ratio must be of a higher tier than

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[7] Ar. Met. 991b.
(Interpreted with the aid of the translation by Ross, 1947, 708, in which the passage is translated as, "Again, if the Forms are numbers, how can they be causes? Is it because existing things are other numbers, e.g. one number is man, another is Socrates, another Callias? Why then are the one set of numbers causes of the other set? It will not make any difference even if the former are eternal and the latter are not. But if it is because things in this sensible world (e.g. harmony) are ratios of numbers, evidently the things between which they are ratios are some one class of things. If, then, this – the matter – is some definite thing, evidently the numbers themselves too will be ratios of something to something else.")

other types of Forms. It may not, however, be the highest of Forms, for we have observed that the Forms of the equal and unequal must exist prior to the Form of Ratio, which itself might even be a ratio of these two Forms. This requirement poses a significant problem for the succession of causes, for these Forms, being prior to the Form of Ratio, can certainly not be ratios themselves. They might be entities prior to number all together, thus usurping numbers from the position of primary causes. Alternatively, they might be numbers which, as an exception, are not ratios themselves, and as such, they might either be rational numbers or irrational numbers, and we shall observe at a later point the results of each possibility.

It may be said, however, that even below the Forms of the equal and unequal, the Form of Ratios should not be the highest and simplest Form of mathematical relation, since it is predicated of a particular type of relation. There must then be a simpler Form still, which is predicated of all Forms of particular mathematical operations. If, on the other hand, the Form of Ratios is itself a ratio, it would follow that it must also be the first ratio, for if it is the cause of ratios, then it is impossible for it to be the effect of any ratios other than itself.[8] This suggestion, however, will prove problematic, for rational numbers are relative, such that a certain numerical value will sometimes be a ratio of other numbers, while in other cases it may be one of the two terms of which another number is a ratio. Even in those situations in which the number is a ratio, it may in one case be the ratio of one pair of numbers, while in a different instance it is a ratio of two other numbers. In other cases still, we do not treat them as being ratios at all, and in these situations they may be multiplied by one number of which they might otherwise be a ratio in order to produce another such number. Thus, they may sometimes be causes of the very same numerical values of which they may at other times be effects. This characteristic would result in the Form of Ratios being a ratio of numbers which exist independently on some occasions but on others as the effects of other numbers, and while independent, it might be a cause of the same numbers of which it would otherwise be an effect. In order to resolve this problem, the two numbers that are the causes of the first ratio and all other ratios must be of values that may in no way be ratios. This purpose may be served by irrational numbers, since by their definition they cannot be accurately expressed in the division of one number by another.

This solution, however, presents a significant dilemma, for we must address the confusion of whether irrational values may be called numbers within the context of Platonic philosophy, while considering the implications of this problem in relation to the doctrine of the Forms. According to Paul Pritchard, a scholar of Platonic mathematical philosophy, it seems, based on Plato's mathematical theories, that irrational numbers cannot be recognized as numbers within the doctrine of the Forms. Pritchard indicates this restriction in controverting the arguments of Taylor, 1934 and Scolnicov, 1971, which

[8] The characterization of the Form of Ratios as a ratio places this Form at the beginning point of a recursive hierarchy of ratios which extends to the lowest tier of existence.

assert that Plato recognized the existence of natural and irrational numbers.[9] Pritchard's refutation of Taylor and Scolnicov is related to his interpretation of the term of ἀριθμός, which, according to his position, implies "a set of things."[10] In Scolnicov's argument, however, it is stated that Plato understood irrational values such as √2 and √3 to be numbers. Regardless of whether or not Plato recognized irrational values as numbers, the conclusion will undermine the position of the Forms as numbers, ratios, and primary causes. If irrational values are not recognized as numbers according to Plato's philosophy, then the irrational values of which the Form of Ratio is a ratio would be either primary causes themselves, or perhaps more plausibly, caused by something entirely prior to quantity. The function of numbers as a primary cause in Plato's doctrine would thereby be refuted, and the integrity of Plato's position thus destroyed. According to Aristotle's evaluation of Plato's doctrine, however, it is possible for something that is not a number to be a ratio of numbers, as is the case with the Form that Aristotle terms as the αὐτοάνθρωπος.[11] If, however, the Form of Ratio is assumed to be a ratio of irrational values, those values cannot be ratios of other numbers. We have thus determined that if irrational values are not considered to be numbers, it would be absurd to treat the Form of Ratio as a ratio of irrational values.

If, however, irrational values are considered to be numbers, there are two possible conclusions, each of which poses a certain problem for the compatibility of numbers and Forms in Plato's philosophy. One might argue, for instance, that irrational values are not only numbers, but are also self-causing, and in this case, there would be no primary cause prior to numbers. As a result, Socrates' attempt in the *Phaedo* to demonstrate the existence of separate substance prior to numbers would be unsuccessful. If, however, irrational numbers are recognized as numbers, and are caused by something else, the thing by which they are caused must be something that is not a number. The position of numbers as the primary cause in Plato's ontology is therefore disproven, once again demonstrating the incompatibility of Form and number as primary causes.

We may avoid these issues by treating the Form of Ratio as a ratio of rational numbers, though we are then faced with the problem concerning the relativity of numbers. Through the use of this approach, the Form of Ratio is a ratio of numbers which cannot be considered ratios in this circumstance, but may be understood as such in all other contexts.

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[9] Paul Pritchard. *Plato's Philosophy of Mathematics*. International Plato Studies 5. Sankt Augustin: Academia Verlag, 1995, 15.
Cf. A.E. Taylor. "Forms and Numbers: A Study in Platonic Metaphysics", *Philosophical Studies* (1934), 102.,
S. Scolnicov. "On the Epistemological Significance of Plato's Theory of Ideal Numbers", *MH* 28 (1971) , 93.
In his work, Pritchard suggests that the majority of scholars on this topic argue that Plato did not consider irrational values to be numbers.

[10] Pritchard, 15.
This definition seems to restrict the concept of ἀριθμός to a dependency on finite objects, contrary to Socrates' purpose as described in the dialogue of the *Phaedo*.

[11] Ar. Met. 991b. 19-21.
"And the Form of man (αὐτοάνθρωπος), whether it is a number (ἀριθμός) of some sort or not, will be the same as a ratio (λόγος) in numbers of certain things, and not a number (ἀριθμός)…"

If this explanation is assumed to be correct, there are two rational numbers which are treated as first causes only insofar as they are the values of which the Form of Ratio is a ratio. Under all other circumstances, however, each of these two numbers may be recognized as an effect of two numbers.[12] For these prior numbers, there may, furthermore, be several possible combinations of values; for example, 3 is understood to be a ratio of 27 to 9, but also of 6 to 2. Thus we observe that one instance of 3 may not be the same as another instance of 3, and that numbers are therefore relative rather than being absolute. This inconsistency invalidates the position that numbers are a first cause, as Aristotle indicates in stating,

> "Overall, the arguments (λόγοι) of the Forms (εἰδῶν) overturn the existence of the things that we desire to exist more greatly than the Forms (ἰδέας). So it follows that number (ἀριθμόν) rather than the Dyad (δυάδα) is the first cause (πρώτην), and that the relative thus precedes the absolute, and in every respect the conclusions following from the assumptions regarding the Forms (ἰδεῶν) are contradictory to the premises (ἀρχαῖς) of the same."[13]

As Aristotle has explained here, the identification of numbers as the first cause will have the consequence that the relative is prior to the absolute, and from this position it follows that the effect is more knowable and more complete in its being than the cause. The notions of the relative and the absolute to which Aristotle makes reference are also clearly perceived within mathematical functions insofar as the value of the independent variable is the cause of the value of the dependant variable, with the absolute term thereby dictating the value of the relative term. The ramifications of numbers being a first cause are therefore, somewhat ironically, inimical to some of the central principles of mathematics.

More significantly, however, it follows that if numbers are to be understood as a cause prior to the Dyad, the Dyad must consequently be confined to mathematical principles. This restriction is more readily apparent in the taxonomy of causes and effects which appears to follow the system of Forms and ratios detailed by Aristotle.[14] According to this system, each sensible object is understood to be ratios of certain numbers, while the Form of which it partakes is considered to be a ratio of other numbers. It is also the case that the numbers of which the sensible object and the Form are ratios, are themselves ratios of other numbers, which are, in turn ratios of other numbers still. Each Form, furthermore,

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[12] The numerical value of 1 is unique in this respect. It behaves in the same manner as all other rational numbers in that its role may change from one mathematical function to another, such that in some circumstances it may be a cause, while in others it is an effect. Aristotle explains, however, that there are several senses of "one", (Ar. Met. 1052a15-1052b1.), and that 1 in the numerical sense is the cause of all other numbers in that it is the unit according to which all other numerical values are determined. (Ar. Met. 1052b. 20-22).

[13] Ar. Met. 990b. 17-23.
(Interpreted with the aid of the translation by Ross, 1947, 706, in which the phrase "καὶ τὸ πρός τι τοῦ καθ' αὑτό" is translated as "that the relative is prior to the absolute." The first sentence as translated here is largely a direct translation, assisted by Ross' translation, which is more idiomatic.)

[14] See citation 4.

partakes of the Form of Ratio, which will be a ratio of certain numbers itself, assuming that it abides by the same principles as all other Forms. The result of this hierarchy, however, is unacceptable due to the absence of a clearly defined point of origin, which leads to a case of infinite regress. Since we have determined that the Forms of ratios is itself a ratio, then it must be the case, as Plato aims to demonstrate in the *Phaedo*, that there must indeed be a first cause prior to number, for there would otherwise be an ἀπορία.

The first strand of this ἀπορία is that not only are all Forms treated as numerical ratios, but the numbers of which they are ratios must themselves be ratios, and the numbers of which they are ratios must be ratios as well. The continuation of this succession results in a state of infinite regress, and is therefore not admissible. The other strand is for the Form of Ratio to violate the doctrine of the Forms by being not a ratio, but a number prior to ratio. In this circumstance, the Forms below the Form of Ratio would be ratios of the Form of Ratio to a certain other number. Within this system, the Form of Ratio, as a universal constant would function as the unifying element of the One. The other number, as a variable with ostensibly infinite possible values, functions in the role of the Dyad. Although this solution addresses the problem of infinite regress, it fails to account for the prior position of the Forms of the equal and unequal, and violates the principles of the One and the Dyad. We have identified the Form of Ratio as a Form, and thus as number, and so to equate it with the One would be a contradiction of Plato's *Parmenides*, wherein the unquantifiable nature of the One is expressed thusly,

> "And there will be no magnitude (μέγεθος) in it, for something, other than magnitude (μεγέθους) itself, will be greater than something else, particularly that into which magnitude (μέγεθος) is placed, and there will however be nothing small which it must exceed, if it is large. This is impossible if there is no smallness (σμικρότης) in anything."[15]

This passage indicates that no relative magnitude is ascribed to the One, clearly demonstrating that the One cannot be reduced to a numerical quantification. The Dyad, moreover, cannot, as Aristotle indicates,[16] be confined to mathematical principles. To be sure, the existence of the Dyad prior to number is necessary for the laws of mathematics as they are known to us, for it is presumably impossible for measurable quantities and operational relations to be causes of the principle of relation.

Aristotle thus demonstrates that although Plato has articulated the causes of being prior to number, he has not entirely succeeded in integrating these principles within his doctrine of the Forms. Due to his association of the Forms with ratios and of numbers with the first cause, Plato's ontology is subject to certain rules which contravene the strictures established by the concepts of the One and the Dyad. The application of these principles is thereby limited to their role in mathematics, and it is for this reason that Aristotle sees in

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[15] Plat. Parm. 150β-ξ.
(Interpretation of passage assisted through translation by Mary Louise Gill and Paul Ryan in Cooper, 1997: 383.)

[16] See citation 9.

Platonic philosophy only a miniscule departure from the tenets of the Pythagoreans. The structural problem of the doctrine of the Forms is initially clear in observing that the Forms of the equal and unequal must be ratios, and therefore they are placed in the impossible position of being prior to a Form upon which their own existence is dependant. Even with this consideration aside, we have determined that the Form of Ratio must be a ratio itself, and unless it may be demonstrated that the numbers of which it is a ratio will not be ratios themselves, the succession of ratios as causes and effects will lead to infinite regress. In order to prevent infinite regress, however, there must be a pair of numbers that will not be ratios themselves, and of which the Form of Ratio will be a ratio. If these numbers are irrational, then either the position of numbers as primary causes will be destroyed, or Socrates' argument in the *Phaedo* will have failed. If, however, these numbers are rational, then that which is relative will be placed in the position of the primary cause, which is absurd. The Form of Ratio, furthermore, cannot be the One as discussed in the *Parmenides*, for not only may it be distinctly defined as one of the Forms, but there are certain things prior to it, namely the Forms of the equal and the unequal. This state, however, is still unacceptable, for there are entities prior to the Form of Ratio which are either ratios themselves, or are prior to the Forms. Thus we observe that Forms and numbers are unable to share in the role of primary causes. One alternative is to regress entirely to the position of the Pythagoreans, such that numbers are primary causes without the inclusion of the Forms, but Aristotle does not follow this path. Rather, from this entangled intermediate state, he shall abandon the notion of numbers as primary causes as he embarks, and thus guides those who will follow him, on a journey towards the understanding of transcendent substance.

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