The Ascending Angle

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In books M and N of the *Metaphysics*, Aristotle's discourse displays a circular return to the refutations of Platonic and Pythagorean mathematical doctrine that initiate the progression of Aristotle's discussion of the science of τὸ ὂν ᾗ ὄν. He returns specifically to the question posed in the twelfth ἀπορία of book B, that is, whether or not mathematical objects such as numbers, points, lines, and planes belong to the category of οὐσίαι. He determines that these objects apparently must be οὐσίαι, and the initial reason that they ought to belong to this category is as follows,

> For if they are not entities [οὐσίαι] it escapes us to say what sort of thing being is and what sorts of things are the entities of being. For accidents, motions, relations, affectations, and ratios do not seem at all to imply entity; for they are ascribed to all things belonging to substratum, and are certainly not discrete objects. These things seem most of all to imply entity, water, earth, fire, and air, from which composite corporeal things are comprised, and from which arise things such as heat, coldness, and objects of experience of this sort, not independent entities, and the corporeal abides alone from the modification as something existing, and as something being an entity. But if indeed the corporeal is subordinate to the entity of manifestation, it will be separate from image, and separate from unit and from point; yet since the corporeal will then destroy these things, and it seems that without these things those belonging to the corporeal will be destroyed, it is impossible for the corporeal to exist without these things.[1]

Thus, as the elements alone do not account for the production of subsistent beings, and corporeal objects cannot subsist independently of lines and units, there must be another factor which is responsible for the existence of things that may be identified with a certain definition. Mathematical objects are likely candidates to serve this purpose, for surely if they did not exist, or did not adhere to the axioms by which they are governed, then οὐσίαι could not exist at all, or not, at any rate, in the manner in which we observe them to do so. It is absurd, however, that those things which are subordinate to mathematical objects should be οὐσίαι, while mathematical objects themselves are not οὐσίαι. It is, however, impossible for them to be οὐσίαι, as Aristotle thus demonstrates,

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[1] Aristotle *Metaphysics* 1001b29-1002a8: "εἰ μὲν γὰρ μή εἰσιν, διαφεύγει τί τὸ ὂν καὶ τίνες αἱ οὐσίαι τῶν ὄντων. τὰ μὲν γὰρ πάθη καὶ αἱ κινήσεις καὶ τὰ πρός τι καὶ αἱ διαθέσεις καὶ οἱ λόγοι οὐθενὸς δοκοῦσιν οὐσίαν σημαίνειν· λέγονται γὰρ πάντα καθ' ὑποκειμένου τινός, καὶ οὐθὲν τόδε τι. ἃ δὲ μάλιστ' ἂν δόξειε σημαίνειν οὐσιαν, ὕδωρ καὶ γῆ καὶ πῦρ καί ἀήρ, ἐξ ὧν τὰ σύνθετα σώματα συνέστηκε, τούτων θερμότητες μὲν καὶ ψυχρότητες καὶ τὰ τοιαῦτα πάθη, οὐκ οὐσίαι, τὸ δὲ σῶμα τὸ ταῦτα πεπονθὸς μόνον ὑπομένει ὡς ὄν τι καὶ οὐσία τις οὖσα. ἀλλὰ μὴν τό γε σῶμα ἧττον οὐσία τῆς ἐπιφανείας, καὶ αὕτη τῆς γραμμῆς, καὶ αὕτη μονάδος καὶ τῆς στιγμῆς· τούτοις γὰρ ὥρισται τὸ σῶμα, καὶ τὰ μὲν ἄνευ σώματος ἐνδέχεσθαι δοκεῖ εἶναι, τὸ δὲ σῶμα ἄνευ τούτων ἀδύνατον." All translations performed by author, assisted with external sources where noted.

> But if the opposite is said, that corporeal structures and points are indeed an entity, so that we do not destroy that which belongs to corporeal things (therefore making it impossible for them to be in sensible objects), there will be no entity at all.[2]

For the purpose of expressing precisely that of which the category of οὐσίαι is comprised, Aristotle has discussed mathematical objects in sufficient detail by stating that they are not οὐσίαι. In order, however, to articulate a science dedicated to the examination of being with respect to itself, more is required than the mere exclusion of mathematical objects from the categories with which they are incompatible. We must be able to construct an exact definition of mathematical objects, and to articulate the manner in which our apprehension of these objects advances our intellect towards comprehension of transcendent entity.

Aristotle advances his discourse toward this objective in books M and N, wherein he provides extremely precise descriptions of the theories held by Platonists and Pythagoreans regarding the nature of mathematical objects, and presents similarly circumspect counterarguments against these positions. These counterarguments function as negations, removing from our consideration of mathematical objects those characteristics which have been incorrectly ascribed to them, thereby allowing us to arrive at a more accurate understanding of the role of mathematical objects in the structure of a complete reality. There are, as Aristotle explains, three possible manners in which numbers might be said to exist, and he thus describes them:

> It is necessary, if indeed there are mathematical objects, that they either exist distinctly within sensible things, just as some state, or that they are separate from sensible things (and indeed some say this), or if neither is the case, then they do not exist at all, or they exist in some other manner.[3]

The first of these suggestions is impossible, for it would lead to the indivisibility of sensible entities, the reason for which Aristotle provides by stating,

> But concerning these it is clear that to divide corporeal things would be impossible, for they would be divided with respect to plane, and plane with respect to line, which would be divided by point, and therefore if it is impossible to divide the point, it is likewise impossible to divide the line, and if it is so with the prior, so it is with that which follows from it.[4]

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[2] *Ibid.*, 1002a15-18: "ἀλλὰ μὴν εἰ τοῦτο ὁμολογεῖται, ὅτι μᾶλλον οὐσία τὰ μήκη τῶν σωμάτων καὶ αἱ στιγμαί, ταῦτα δὲ μὴ ὁρῶμεν ποίων ἂν εἶεν σωμάτων (ἐν γὰρ τοῖς αἰσθητοῖς ἀδύνατον εἶναι), οὐκ ἂν εἴη οὐσία οὐδεμία."

[3] Aristotle *Metaphysics* 1076a33-8: "Ἀνάγκη δ', εἴπερ ἔστι τὰ μαθηματικά, ἢ ἐν τοῖς αἰσθητοῖς εἶναι αὐτά, καθάπερ λέγουσί τινες, ἢ κεχωρισμένα τῶν αἰσθητῶν (λέγουσι δὲ καὶ οὕτω τινές)· ἢ εἰ μηδετέρως, ἢ οὐκ εἰσὶν ἢ ἄλλον τρόπον εἰσιν."

[4] *Ibid.*, 1076b4-9: "ἀλλὰ πρὸς τούτοις φανερὸν ὅτι ἀδύνατον διαιρεθῆναι ὁτιοῦν σῶμα· κατ' ἐπίπεδον γὰρ διαιρεθήσεται, καὶ τοῦτο κατὰ γραμμήν, καὶ αὕτη κατὰ στιγμὴν, ὥστ' εἰ τὴν στιγμὴν διελεῖν ἀδύνατον, καὶ τὴν γραμμήν, εἰ δὲ ταύτην, καὶ τἆλλα."

As Aristotle informs us, it is similarly impossible for mathematical objects to exist separately from sensible entities. If they were to exist in such a way, then there would be several redundant classes of mathematical objects beyond the sensible, such that in addition to sensible mathematical objects there would exist another class of solids, three classes of planes, four classes of lines, and five classes of points. Regarding this hierarchy, Aristotle comments, quite correctly, that it would impossible to determine which of these classes are the objects of mathematical science.[5] Though mathematical objects are not capable of existing as entities separate from sensible things, we may nevertheless say with certainty that mathematical objects are necessary for the existence of sensible things according to their definition. We may therefore infer that mathematical objects are prior to the sensible in some respect, and Aristotle discusses the manner of this priority by stating,

> For it is necessary, based on this manner of existence, for these things to be prior to sensible magnitudes, while in truth they are posterior, for unrealized magnitude is prior in generation, but is posterior in entity, being lifeless separate from what is alive.[6]

It would be most accurate to say, however, that while mathematical objects are in this prior state, the potentiality of which Aristotle speaks does not belong properly to the mathematical objects, but to the sensible entity in whose generation they are involved. Indeed, it would be absurd to claim that the mathematical objects themselves are in potency, since they not only serve an instrumental purpose in the generation of the sensible entity, but following generation they persist in maintaining the adherence of the entity to the parameters of its definition. If indeed any sort of existence may be ascribed to mathematical objects, then it might plausibly be said that their action is their existence, for indeed they do not exist apart from their purpose of comprising the structure of sensible entities. They might most accurately be regarded as specific instances of universal mathematical functions which are not entities, yet are immutable, and therefore

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[5] *Ibid*., 1076b25-36 (interpretation assisted by translation of Armstrong and Tredennick, 1935). In this hierarchy, Aristotle counts among the separate mathematical objects not only those which are presumed to exist unto themselves, but also those contained within other separate mathematical objects. For instance, the separate lines contained in one of the classes of separate solids would be counted as one of these classes. Though we are not able to determine which of the separate mathematical objects would be the objects of mathematical science, we may be certain that no science can pertain to sensible mathematical objects, since these are not the most knowable. Concerning separate points, a further problem, although Aristotle does not discuss it, is that in order for them to have any significance, they must hold a definite value in relation to a certain axis, and there must therefore be separate lines even prior to the first separate points, and prior to these there must be other separate points, prior to which there would be other separate lines. From this hierarchy there will therefore result a state of infinite regress in which it will be impossible for anything to exist.

[6] *Ibid*., 1077a16-20 (interpretation assisted by translation of Armstrong and Tredennick, 1935): "ἀνάγκη γὰρ διὰ τὸ μὲν οὕτως εἶναι αὐτὰς προτέρας εἶναι τῶν αἰσθητῶν μεγεθῶν, κατὰ τὸ ἀληθὲς δὲ ὑστέρας· τὸ γὰρ ἀτελὲς μέγεθος γενέσει μὲν πρότερόν ἐστι, τῇ οὐσίᾳ γ’ ὕστερον, οἷον ἄψυχον ἐμψύχου."

supremely intelligible. We have therefore identified mathematical functions as a category of object which necessarily exists in the divine νοῦς as a first principle for the quantitative algorithms responsible for ordering and maintaining the structure of a complete reality.

Out of all the functions responsible for the architecture of existence, it may reasonably be said that the most universal are those pertaining to the behaviour of numbers on an arithmetic level. Aristotle confirms this suggestion in the first book of the *Metaphysics*, wherein, concerning the hierarchical relation of the arithmetic and geometrical sciences, he states,

> The first of the sciences are those which are the most exact, for the sciences belonging to those that contain less are more exact than the things said of that which is added, as arithmetic is more exact than geometry.[7]

In stating that the most exact sciences are those in which the least is contained, Aristotle demonstrates that the most exact are the simplest insofar as composition is ascribed to them to the least extent of all sciences. Concerning composition, it is already apparent to us that those things which are simple are prior to those which are more composed, since those things which are composed are caused by that which comprises them. We will also note that without the arithmetic operations being as they are, the science of geometry would be destroyed, and the science of astronomy would therefore be impossible as well. If, for instance, the concept of arithmetic multiplication did not exist, or functioned in a manner different from that by which it is characterized, then the computation of the area of a plane or the volume of a geometric solid would be impossible. There are, furthermore, certain functions governing events in the natural world which may be said to have a precise domain and range, that is to say, a dependant and independent variable, respectively. We know, for example, that the amount of moisture which may be contained within the atmosphere without the occurrence of precipitation is a function of the temperature of the air. This temperature, moreover, is by no means a primary domain, since, among other variables, it is a function of the position of the earth relative to the sun with respect to the geographic location in question, as well as the measure of intrusive gaseous matter contained within the atmosphere. These too are functions of other mathematical variables, yet these functions are but a few in a vast, intricate network of operations by which the first νοῦς presides over the activity of reality. If, therefore, the simplest arithmetic functions did not exist in νοῦς, reality would not adhere to any order, and nothing would exist except by chance.

Although the arithmetic science is considered to be more exact than other mathematical sciences, and is necessary for the correct apprehension of all others, it is not

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[7] Aristotle *Metaphysics* 982a26-8 (interpretation assisted by translation of Tredennick, 1933): "ἀκριβέσταται δὲ τῶν ἐπιστημῶν αἳ μάλιστα τῶν πρώτων εἰσίν· αἱ γὰρ ἐξ ἐλαττόνων ἀκριβέστεραι τῶν ἐκ προσθέσεως λεγομένων, οἶον ἀριθμητικὴ γεωμετρίας."

necessarily the noblest. It may be said, for instance, that although the λόγος of geometry does not enable us to construct a complete articulation of existence, it allows us to express considerably more than might be conveyed if we possessed only a λόγος of arithmetic. In order, therefore, to apprehend reality to an extent closer to its totality, and therefore elevate our activities of thinking and being nearer to that of the divine, we must also avail ourselves of the λόγος of geometrical science. Aristotle's denial of separate existence from the objects of geometry may appear to diminish their ontological position, but careful consideration will reveal to us that Aristotle is in fact elevating these objects to the rank of their proper significance. The comment presented by Stewart Shapiro in *Thinking About Mathematics* expresses this ἀλήθεια by suggesting that the existence of geometrical objects as distinct entities "would sever the tie with observed objects."[8] This explanation indicates that the contemplation of geometrical objects as independently existing entities diminishes the significance of these objects in contrast to their true purpose in the structure of existence. The suggestion that they exist independently implies that they are capable of existing entirely at rest and without purpose. Through their presence in sensible things, geometrical objects are perpetually at work, fulfilling a vital purpose in the generation and maintenance of the structure of αἰσθητοί.

Indeed, the observation of geometrical objects at work within the tier of the sensible is crucial to our understanding of the functions and axioms which constitute the λόγος of the geometrical science. Aristotle illustrates this connection at the beginning of the *Mechanical Problems* by stating,

> These things are not entirely the same as natural problems though not entirely different, but are common between the mathematical and natural observations, for just as the means is visible through mathematics, the function is observable by way of physics.[9]

This statement indicates that if we possess an apprehension of the axioms pertaining to mathematical objects, we will be capable of inferring ἀλήθειαι regarding certain characteristics and behaviours attributable to sensible objects without necessarily witnessing such qualities and actions through sensory observation. Though these mathematical inferences are separate in thought, it is in the generation and movement of the objects of physics that they participate in reality, and they are therefore inseparable from the objects of geometry when they are considered in an active state rather than from an abstract standpoint. If our apprehension of these objects of geometry is to be of the nearest possible similitude to the manner in which they are articulated within the first νοῦς, it behoves us to address the question of which type of geometrical object is of the highest standing in terms of its role in generation and motion. In discussing the noblest of

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[8] Stewart Shapiro, *Thinking About Mathematics: The Philosophy of Mathematics* (Oxford: Oxford University Press, 2000), 71.

[9] Aristotle *Mechanical Problems* 847a24-9 (interpretation supplemented with translation by Hett, 1936): "ἔστι δὲ ταῦτα τοῖς φυσικοῖς προβλήμασιν οὔτε ταὐτὰ πάμπαν οὔτε κεχωρισμένα λίαν, ἀλλὰ κοινὰ τῶν τε μαθηματικῶν θεωρημάτων καὶ τῶν φυσικῶν· τὸ μὲν γὰρ ὥς διὰ τῶν μαθηματικῶν δῆλον, τὸ δὲ περὶ ὃ διὰ τῶν φυσικῶν."

geometrical objects, Aristotle is of great assistance to us, for he informs us explicitly, “The circle holds the principle of the cause of all of these things.”[10] It should come as no surprise to us that the circle contains the foundations of the mathematical concepts related to physical motion, for as we shall soon observe, the circle is the source of all geometrical principles, as Aristotle demonstrates in book Z of the *Metaphysics*, in which he states,

> The definition of a circle does not contain that of its partitions, while the principle of the syllable contains that of its elements, however the circle is divided into its partitions just as the syllable is divided into its elements.[11]

The salient distinction between these two types of divisions, however, is that the circle is not composed of its partitions, which are therefore derivatives of the whole. Aristotle demonstrates this priority further at a later position within the same book, at which point he explains,

> The circle and semicircle behave similarly, for the semicircle is divided from the circle, and the finger is divided from the whole [man].[12]

Since the circle is understood to be a cause of all partitions thereof, including the semicircle, we may then correctly say that the circle is the source of all principles which constitute the axioms of geometrical science. It is due to the axioms and functions associated with the measurements of the circle that we are able to make any precise statements regarding the structure of angles, planes, and solids.

Since Aristotle has demonstrated to us that the circle is the most universal of all of the objects of geometry, there remains one final consideration. We must be able to explain how the apprehension of this most universal of geometrical objects may enable knowledge of transcendent entity. As we shall soon observe, the geometrical science serves this purpose by allowing knowledge of the motion of the heavenly spheres, and although, as with geometry, the astronomical science is less exact than arithmetic, and seemingly even less so than geometry, it is nonetheless more noble, for, as Aristotle explains in book Λ of the *Metaphysics*,

> Out of the myriad of mathematical sciences that we must study, the nearest in object to philosophy is that of astronomy, for this alone carries out the examination of entities that are both sensible and imperishable, and the others are not at all concerned with entities, as is the case with those dedicated to numbers and the objects of geometry.[13]

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[10] *Ibid*., 847b16-17: "Πάντων δὲ τῶν τοιούτων ἔχει τῆς αἰτίας τὴν ἀρχὴν ὁ κύκλος."
[11] Aristotle, Books 1-9, 1034b25-8: "τοῦ μὲν γὰρ κύκλου ὁ λόγος οὐκ ἔχει τὸν τῶν τμημάτων, ὁ δὲ τῆς συλλαβῆς ἔχει τὸν τῶν στοιχείων· καίτοι διαιρεῖται καὶ ὁ κύκλος εἰς τὰ τμήματα ὥσπερ καὶ ἡ συλλαβὴ εἰς τὰ στοιχεῖα."
[12] *Ibid*., 1035b9-11: "ὁμοίως δὲ καὶ ὁ κύκλος καὶ τὸ ἡμικύκλιον ἔχουσιν· τὸ γὰρ ἡμικύκλιον τῷ κύκλῳ ὁρίζεται, καὶ ὁ δάκτυλος τῷ ὅλῳ."
[13] Aristotle *Metaphysics* 1073b4-9: "τὸ δὲ πλῆθος ἤδη τῶν φορῶν ἐκ τῆς οἰκειοτάτης φιλοσοφίᾳ τῶν μαθηματικῶν ἐπιστημῶν δεῖ σκοπεῖν, ἐκ τῆς ἀστρολογίας· αὕτη γὰρ περὶ οὐσίας αἰσθητῆς μὲν ἀϊδίου δὲ ποιεῖται τὴν θεωρίαν, αἱ δ’ ἄλλαι περί οὐδεμιᾶς οὐσίας, οἷον ἥ τε περὶ τοὺς ἀριθμοὺς καὶ τὴν γεωμετρίαν."

Although the mathematical sciences dedicated entirely to numbers and to the objects of geometry do not treat entities directly within the scope of their analysis, they are nonetheless vital in the apprehension of the existence and motion of celestial objects. It is therefore in this respect that the mathematical sciences enable our apprehension of transcendent entities.

We shall now engage, albeit briefly, in an attempt to treat the sensible yet imperishable entities of the heavens by means of an astronomical model which is described thus by Aristotle and attributed to Eudoxus, who

> placed the orbit of each of the sun and the moon to be in three spheres, the first of which is that of unmoving stars, the second of which is that which passes through the central stars of the Zodiac (translated by Armstrong and Tredennick as "the circle which bisects the Zodiac), while the third is that which is placed according to the slanting in the breadth of the Zodiac. The sphere upon which the moon travels is slanted at a higher angle than that upon which the sun travels.[14]

This description is not exhaustive, though it provides us with sufficient information for the purpose of articulating Eudoxus' concept of the orbit of the sun and moon in relation to Earth with respect to the passage of days and years. In *Early Physics and Astronomy*, Olaf Pedersen provides a precise quantitative synopsis of Eudoxus' concentric sphere model. Pedersen describes the three spheres of lunar rotation by explaining that the outermost sphere represents the daily orbital path of the moon,[15] and he informs us at an earlier point in this work that a sidereal day consists of $23^h\ 56^m$.[16] We may therefore calculate that a sidereal day is comprised of 1,436 minutes. By dividing this measurement by 360°, we will be able to compute the number of minutes allotted to each degree of the outermost orbit, and from this operation we shall determine that each degree consists of $3.9\overline{8}$ minutes. Pedersen explains that the middle sphere undergoes a full rotation relative to the outermost sphere over a period of 223 synodic months.[17] We are further told by Patrick Moore and Robin Rees in *Patrick Moore's Data Book of Astronomy* that a synodic period is "the interval of time between successive new moons or successive full moons," and that each such period consists of $29^d\ 12^h\ 44^m$,[18] in contrast to the

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[14] *Ibid*., 1078b17-23 (interpretation supplemented with translation by Armstrong and Tredennick, 1935). Εὔδοξος μὲν οὖν ἡλίου καὶ σελήνης ἑκατέρου τήν φορὰν ἐν τρισὶν ἐτίθετ' εἶναι σφαίραις, ὧν τὴν μὲν πρώτην τὴν τῶν ἀπλανῶν ἄστρων εἶναι, τὴν δὲ δευτέραν κατὰ τὸν διὰ μέσων τῶν ζῳδίων, τὴν δὲ τρίτην κατὰ τὸν λελοξωμένον ἐν τῷ πλάτει τῶν ζῳδίων· ἐν μείζονι δὴ πλάτει λελεξῶσθαι καθ' ὃν ἡ σελήνη φέρεται ἢ καθ' ὃν ὁ ἥλιος. According Armstrong and Tredennick (1935: 156, n.1), Eudoxus was born in Cnidus and lived circa 408-355 BC. Tredennick and Armstrong state that Eudoxus "was a pupil of Plato and a distinguished mathematician."

[15] Olaf Pedersen, *Early Physics and Astronomy: A Historical Introduction*, revised ed. (Cambridge: Cambridge University Press, 1993), 64.

[16] *Ibid*., 53.

[17] *Ibid*., 64.

[18] Patrick Moore and Robin Rees, *Patrick Moore's Data Book of Astronomy*, (Cambridge: Cambridge University Press, 2011), 25.

measurement of a synodic month in Eudoxus' model, which consists of 29.53 days,[19] or $29^d\ 12^h\ 43^m\ 12^s$. In order to determine the number of synodic months allotted to each degree of the median sphere, we will divide 223 synodic months by 360°, and this operation will produce a result of $0.619\overline{4}$ synodic months. Although such calculations do not necessarily suggest any clear purpose in themselves, they are nevertheless invaluable for the most precise possible examination of hypothetical astronomical models such as that proposed by Eudoxus. Through these inferences, it may be possible to verify the accuracy of such models in relation to our observations of the cycles of the passage of days. Plato states in the *Timaeus* that through mastery of these calculations, we bring the cosmic orbits within ourselves into accord with those of the Demiurge.[20] In this regard, there appears, between Plato and Aristotle, a common position that through precise mathematical analysis of the firmament, we ascend closer to transcendent νοῦς, which, as we shall soon observe, may be understood according to Aristotle's position as the origin of the formulae that we have considered.

Through examination of the impossibilities concerning the nature of the objects of mathematics, we have constructed a viable explanation for their proper place within the structure of reality, having determined that they are specific results of the universal functions which ensure the adherence of all entities to the axioms governing the framework of existence. Just as the objects of mathematics, though not entities in themselves, are inextricably connected to sensible entities, the mathematical functions, though they are not independent entities, are inexorably bound to the intelligible principles governing the structure and activity of sensible beings. As Thomas Aquinas indicates, intelligible principles belong to transcendent intellect, since the transcendent intellect cannot be said to possess knowledge according to ideas, except insofar as those ideas are within it.[21] The laws and formulae of mathematics must therefore exist as known within the divine νοῦς, and it is through the application of these functions that there exists order within reality. It is therefore by apprehension of these functions that we are able to possess knowledge of the algorithms that constitute this order, and so do we peer ever so slightly into the light of the first intellect.

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[19] Pedersen, 64.

[20] Plato, *Timaeus*, 47c.

[21] St. Thomas Aquinas, *The Summa Theologiae of St. Thomas* Aquinas, vol. 1, Latin-English Edition, trans. The Father of the English Dominican Province (Scotts Valley: CreateSpace, 2008), I, q. 15, a. 1, ad 1.