The Root of All Dimensions: Examining Plato's Teachings on the Mathematical Foundations of Knowing and Being

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Chapter 1: Introduction

As beings defined by our faculty for rational articulation of reality, it belongs to us to apprehend the code of laws governing the system to which we belong, that we ourselves might function in the best possible manner within that structure. In seeking to grasp the truths of the cosmos, our approach to the explanation of the structure and activity of the universe is to describe it in terms of the rules of mathematics. In what manner, however, are we to define these mathematical principles? Are they mere constructs of the human imagination, or do they rather represent a system of regulations belonging properly to reality? If, moreover, the laws of mathematics constitute operative patterns within reality, does our use of mathematical principles in the explanation of the cosmos reflect an essential connection between the human intellect and the structure of the natural world as a whole? In addressing such questions as these, we advance toward a greater capacity for inquiry regarding the structure of the cosmos, as well as the proper place of our species within that system.

We must first, however, address the problems with the thesis that mathematics is a contrivance of the human imagination. As explained by Shapiro (1997), this position is asserted by the mathematical philosophy of intuitionism, a branch of the anti-realist school which rejects the law of excluded middle[1] (and thus rejects the binary truth or falsehood of all mathematical statements) on the basis that "These methodological principles are symptomatic of faith in the transcendental existence of mathematical objects or the transcendental truth of mathematical

1. Aristotle, *Metaphysics*, IV. 1011b23-24. Aristotle describes in this passage the law of excluded middle by stating that "No truth can be allowed between two opposite statements, but rather it is necessary to affirm one of the possibilities and deny the other." ἀλλὰ μὴν οὐδὲ μεταξὺ ἀντιφάσεως ἐνδέχεται εἶναι οὐθέν, ἀλλ᾽ ἀνάγκη ἢ φάναι ἢ ἀποφάναι ἓν καθ᾽ ἑνὸς ὁτιοῦν. All translations are my own unless otherwise stated.

statements."[2] The intuitionist argument therefore places our knowledge of the universe, and thus our role within it, in a particularly vulnerable position. If the mathematical foundations of scientific theories are trivialized, then it becomes largely impossible for us to construct any meaningful articulation of the universe.

Even without the rejection of the law of excluded middle, the treatment of mathematical principles as human constructions proves to be problematic in several regards. It devalues our activity of scientific investigation to a task of imposing our vision of order upon a world that we perceive as being otherwise chaotic and devoid of rational structure. This approach places us in the uncomfortable position of mastery over that which we examine, as opposed to one in which we recognize our place as self-aware components, or cognizant cogs, within the system to which we belong. It also fails to account for the natural properties of the human intellect that incline it towards inquiry into the nature of reality; for it treats mathematics, and by extension, all of science, as nothing more than a product of the human imagination. In this respect, the anti-realist position creates an inconsistency within itself; for in attempting to argue that scientific principles are mere projections of the human mind upon the observed natural world, we necessarily assume the givenness of human thought; for otherwise we must even treat the anti-realist position as a mere contrivance of imagination.

We have, furthermore, shown ourselves to be ill-suited for the mastery of the natural world that would be bestowed upon us by the anti-realist argument, as we have all too frequently failed to grasp the ineluctable truth of the finite magnitude and multiplicity which belongs necessarily to tangible objects – a truth which ought to indicate to us that the laws of mathematics constitute a real governing force within the cosmos, and that it therefore behooves us to apprehend the laws, such that we may understand their relevance to us, and use them to properly guide the direction of our existence. To be sure, however, these arguments against the anti-realist position of mathematical philosophy are not intended to show that anti-realism is incorrect, but rather that it may be inadvisable for us to assume that it is correct. Similarly, our consideration of mathematical realism will not demonstrate that the realist position is correct, but rather will present a possible explanation for how it might be correct. Throughout the course of our investigation, any statements that we present as truth are to be understood as such

2. S. Shapiro, *Philosophy of Mathematics: Structure and Ontology* (New York and Oxford: Oxford University Press, 1997), 23.

only within the context of our proposed solution to the disjunction of ontology and epistemology. It may indeed be impossible for us to determine conclusively which of these two positions is correct; for as long as the sensible world as we observe it is assumed to be real, the correctness of either argument is possible; and due to our confidence in sensible reality over the authority of rational argumentation, there is no argument that can speak to the truth of sensible reality.

The consideration of mathematical principles as operative within reality is central to Platonist mathematical philosophy, which, as we shall later observe, may be considered erroneous in terms of its faithfulness to Plato's philosophy. As Balaguer (1998) explains, mathematical Platonism maintains that mathematical objects, including numbers, "…are non-spatiotemporal and exist independently of us and our mathematical theorizing…"[3] Balaguer also notes, however, that according to Benacerraf's argument from the causal theory of knowledge, or CTK, the truth of mathematical Platonism makes it impossible for us to attain knowledge of mathematics. CTK maintains that in order for a particular person to possess knowledge of a certain object or principle, the former and the latter must be "causally related" to one another "in an appropriate way." Benacerraf concludes that if mathematical objects exist outside of the spatiotemporal realm, they are not causally related to humans, and that if, therefore, mathematical Platonism holds true, it is impossible for us to possess mathematical knowledge.[4] To be sure, if mathematical objects are not causally related to the spatiotemporal realm in any regard, then it follows that they have no bearing on the structure and motion of tangible entities, and since they would, in this case, have no relevance to our understanding of mathematics, they would be utterly without purpose.

It is possible for us to solve this problem, while still maintaining

3. M. Balaguer, *Platonism and Anti-Platonism in Mathematics* (Oxford: Oxford University Press, 1998), 5.

4. Balaguer, *Platonism and Anti-Platonism in Mathematics*, 22. See also K. Dorter, *Form and Good in Plato's Eleatic Dialogues: The Parmenides, Sophist, Theaetetus, and Statesman* (Berkeley and Los Angeles: University of California Press, 1994), 39, 41. Op. cit. Plato, *Parmenides*, 133a-c,124b-c. In the passage quoted by Dorter, the problem presented is that if the Forms are entirely separate from sensible reality in their existence, it would be exceedingly difficult to demonstrate that the separate existence of the Forms does not preclude our knowledge of them. Indeed, one of the arguments against our knowledge of the Forms is that they are not within us in any respect (Plato, *Parmenides*, 134b-c). As we shall observe later, however, the nature of the presence of the Forms within the human intellect is of critical importance in explaining our knowledge of them.

the reality of mathematical principles, if we are able to explain some manner in which the separation of mathematical objects from the spatiotemporal world need not preclude their causal relation to tangible beings. A variation of this solution is proposed by Gödel, who suggests that the human intellect attains knowledge of the objects of mathematics by means of "mathematical intuition." The problem with Gödel's position, according to Balaguer, is that it does not account for the assumed lack of causal relation between the objects of mathematics and the spatiotemporal realm. A possible Platonist counter-argument, Balaguer explains, is that the intellect is non-spatiotemporal. He rejects this postulation, however, despite giving little support for doing so, and states that even the identification of the intellect as non-spatiotemporal does not necessarily imply that the mind communicates with the objects of mathematics.[5] There is also, however, no reason to assume that the separation of an object from the spatiotemporal realm must necessarily prevent the object from being causally related to the latter. Indeed, it may be the case that such an unjustified assumption must be put aside in order for a solution to the ontological-epistemological conflict of mathematical realism to be possible.

Chapter 2: Εἶδος

We shall consider such a solution through our examination of the true definition and function of mathematical principles within the philosophical teachings of Plato. Our inquiry shall draw primarily on the *Timaeus*, supplemented by other Platonic texts such as the *Republic* and the *Phaedo*. Proceeding from the doctrine of an eternal, unchanging model as the origin of all knowledge and existence, and the image of perfection to which all things seek to return, we observe that the principles of mathematics, from the foundational unit concept, to the relations of geometric structure, to functions of vast complexity, constitute nothing less than the language of reality itself. Such laws, as we shall see, are dictated by the requirements defined within the schematics of the cosmos, while also governing the structure of these schematics. Amongst themselves, the laws of mathematics must also serve a

5. Balaguer, *Platonism and Anti-Platonism in Mathematics*, 25-26. Op. cit. J. Katz, *Language and Other Abstract Objects* (Totowa, New Jersey: Rowman and Littlefield, 1981), 201. Balaguer maintains that the communication of information between two objects is an activity belonging purely to the physical realm. On a general note, one of the primary weaknesses in his argument against mathematical Platonism is that it rests on the assumption of a purely tangible reality.

mutually defining role towards one another, with rudimentary operations constituting the foundations of complex operations, which in turn dictate the functionality of rudimentary functions. In this regard, the laws of mathematics function as governing principles within the intelligible, as the intelligible is with respect to the tangible; for in their capacity as the patterns by which the intelligible is the highest perfection of all existence, they constitute the foundation by which all order within the intelligible is defined.

In order to discover the precise definition of mathematical principles within Plato's philosophy, we must first establish the specifications that describe all of the objects of knowledge, including the principles of mathematics. These rules are as follows: (1) The Demiurge (the divine craftsman discussed in the *Timaeus*) is understood to be good, and the cosmos to be beautiful.[6] From this requirement, it follows (2) that the cosmos is constructed according to an eternal principle.[7] From this rule, it follows, in turn, that (3) the model that constitutes this eternal principle is entirely unchanging, or "always in the same way."[8] This requirement dictates (4) that the model described can never be in an incomplete state; and from this law, it follows (5) that within the cosmic model there can be no temporal succession; for the immediate and eternal completion and perfection of the cosmic model precludes the possibility that it is subject to any process of construction. All cases of dependency within the model must then be mutual, such that any definitions contained within the model are completely simultaneous insofar as there is no priority or posterity in their relation to one another.

It seems also (6) that there must indeed be schematic definitions of some sort contained within the cosmic model, as Plato characterizes the inquiry occurring in the *Timaeus* as a consideration of "how the framework of models brought itself to perfection…"[9] In mentioning a framework of models, Plato may be referring to the model of the cosmos, or to the sensible cosmos itself (or perhaps to both), yet in

6. Plato, *Timaeus*, 29a2-3. εἰ μὲν δὴ καλός ἐστιν ὅδε ὁ κόσμος ὅ τε δημιουργὸς ἀγαθός.

7. Plato, *Timaeus*, 29a3-6. δῆλον ὡς πρὸς τὸ ἀίδιον ἔβλεπεν: εἰ δὲ ὃ μηδ᾽ εἰπεῖν τινι θέμις, πρὸς γεγονός. παντὶ δὴ σαφὲς ὅτι πρὸς τὸ ἀίδιον: ὁ μὲν γὰρ κάλλιστος τῶν γεγονότων, ὁ δ᾽ ἄριστος τῶν αἰτίων.

8. T. A. Blackson, *Inquiry, Forms, and Substances: Studies in Plato's Metaphysics and Epistemology*, Philosophical Studies Series 62 (Dordrecht, The Netherlands: Kluwer Academic Publishers, 1995), 133. Op. cit. Plato, *Timaeus*, 27d5-28a4. ἀεὶ κατὰ ταὐτὰ ὄν.

9. Plato, *Timaeus*, 28c5-29a1. πρὸς πότερον τῶν παραδειγμάτων ὁ τεκταινόμενος αὐτὸν ἀπηργάζετο.

any case, it is reasonable to suggest that the schematic models of which we have spoken are defined within the great cosmic model; for if the objects of the sensible are models, then each one must be a model of something. It seems also to be the case (7), according to Plato's position, that all knowledge belongs to a single structure; for Socrates states in the *Theaetetus*, to which Crombie (1963) directs our attention, that knowledge is "*not many things, but one.*"[10] The statement cited by Crombie need not necessarily imply that all knowledge belongs to a single structure, and may merely indicate that there is only one definition of knowledge. Crombie appears, however, to interpret the statement as an indication of the essential unity of all knowledge, as he explains that Socrates is, in this case, refuting Theaetetus' assumption of knowledge having some sort of range. In part of the passage of the *Theaetetus* that is cited by Crombie, Theaetetus characterizes such disciplines as geometry, as well as various fields of craftsmanship, as being encompassed within knowledge, using such terms as "all" and "each" to identify each discipline individually.[11] Socrates responds to Theaetetus by stating that the latter is "asking one thing to be given as many," and for a "patchwork" or "quilt" (ποικίλα) instead of a "single stretch of fabric"(ἁπλοῦ).[12] The language used by Socrates in this case suggests not only a unified structure for all knowledge, but a framework that is unified in a simple manner, insomuch as it is not composed of disparate pieces brought together, but rather is properly one, with each portion of it being essentially connected to all others.

It also stands to reason (8) that the single system of all knowledge is the cosmic model according to which the Demiurge constructs the sensible universe, as the belonging of all knowledge to a single system dictates that this system must contain knowledge regarding the architecture of the universe at all levels; and as we have observed before, the sensible cosmos (and presumably, the cosmic model as well, as there is no clear justification for the cosmos possessing non-accidental qualities absent from the model on which it is based) constitutes a framework of models, or τῶν παραδειγμάτων ὁ τεκταινόμενος; and as such, it would

10. I. M. Crombie, *An Examination of Plato's Doctrines*, Volume II: *Plato On Knowledge and Reality* (London: Routledge and Kegan Paul Ltd., 1963), 368. Op. cit. Plato, *Theaetetus*, 146-7.

11. Plato, *Theaetetus*, 146c8-d2. γεωμετρία τε καὶ ἃς νυνδὴ σὺ διῆλθες, καὶ αὖ σκυτοτομική τε καὶ αἱ τῶν ἄλλων δημιουργῶν τέχναι, πᾶσαί τε καὶ ἑκάστη τούτων, οὐκ ἄλλο τι ἢ ἐπιστήμη εἶναι.

12. Plato, *Theaetetus*, 146d3-4. ἓν αἰτηθεὶς πολλὰ δίδως καὶ ποικίλα ἀντὶ ἁπλοῦ

seem that any non-accidental truth pertaining to the tangible world ought to originate in the schematic according to which the tangible world is constructed. From (7) and (8), it follows that the objects of mathematical knowledge belong to the same foundation as all other objects of knowledge, and that the schematic to which they belong is the eternal model of the cosmos.

Based upon (4) and (5), and therefore upon (1), (2), and (3) as well, we may infer (9) that all mathematical laws and formulae governing the cosmos are present immediately and eternally. It also follows (10) that all functional dependencies of mathematical principles upon one another, and of other paradigms upon mathematical principles, are mutual, such that all paradigms, whether they are laws of mathematics or schematics of a certain type of sensible entity, are conditioned by all other paradigms with which they share an essential connection. Thus, with respect to the intelligible definitions of sensible entities, including various species of flora and fauna, the formulae responsible for governing their properties of dimension and motion are dictated by the same mathematical laws whose formulation they define. That is to say, just as the paradigms that govern the tangible are responsible for dictating what is required of the laws of mathematics, the laws of mathematics are similarly the specifications contained within the paradigms governing the tangible realm.

From the mutual relation of all mathematical principles to one another, we also observe indications that the principles of mathematics constitute a single intelligible structure. The inclusion of such a structure within Plato's thought is supported by (7); for according to Socrates's description of the structure of knowledge as ἁπλός, it stands to reason that the non-composed unity of the entirety of knowledge must also translate into all areas of the ἁπλός which are delineated (perhaps erroneously, as we shall consider later) into distinct disciplines by human conventions of scholarly inquiry. Consequently, it would seem that this same ἁπλός unity must be a property by which the laws of mathematics represent a single vast system. As we shall determine later, the ἁπλός unity of the principles of mathematics is of crucial importance in Plato's explanation for the manner in which the human intellect apprehends mathematical principles, which is, in turn, vitally significant in describing the ontological connection of mathematical principles to the structure and operation of the natural world.

First, however, we examine the essential connection of

mathematical paradigms as a singular intelligible structure; for this ἁπλός unity is of crucial importance in explaining the activity by which mathematical principles are apprehended by the human intellect. This activity, in turn, is of vital significance in investigating the causal connection of the laws of mathematics to the realm of the tangible and to the articulation of the natural world by the human intellect. According to the absence of temporal succession from the cosmic model as discussed in (5), it stands to reason (i) that all whole number values and fractions thereof are instantaneously defined according to the intelligible definition of the unit value, and (ii) that the definition of the geometric point accounts for all dimensional levels. According to both (4) and (5), it must be the case that just as the unit principle defines all multiplicities and fractions of 1, the intelligible principle associated with the geometric point must, it would seem, allow for an infinite number of axis variables, and therefore account for all levels of dimensional structure.

In order to explain the relation of the unit principle to the geometric point principle agency of ἀριθμοί within the structure and operation of the cosmos, we shall look to the principles of number theory that have been credited to the Pythagoreans, for although we face considerable difficulty in identifying them conclusively with their supposed contributions to mathematics, their observations regarding the properties of ἀριθμοί may provide us with more profound insight into the intellectual foundations of Plato's mathematical thought. These teachings, furthermore, may represent our best hope for tracing the mathematical philosophy of Plato to its origins, for a multitude of sources Ancient and contemporary suggest connections between Plato and the first Pythagorean order. Plato is widely regarded to have received some of his teachings under the instruction of the Pythagoreans. In the *Metaphysics*, Aristotle notes that Plato's principle of participation in Forms is almost identical to the Pythagorean concept of imitation of numbers, differing only in terminology.[13] Aristotle's criticism in this instance may have been partially correct, though not necessarily in the manner that he intended. In the *Timaeus*, Plato presents the following account of the foundational principles of spatial dimension,

> It needed to be exactly corporeal, visible, and tangible, so he separated fire, and yet even then it was not visible,

13. Aristotle, *Metaphysics*, I. 6. 987b11-13. τὴν δὲ μέθεξιν τοὔνομα μόνον μετέβαλεν: οἱ μὲν γὰρ Πυθαγόρειοι μιμήσει τὰ ὄντα φασὶν εἶναι τῶν ἀριθμῶν, Πλάτων δὲ μεθέξει, τοὔνομα μεταβαλών.

> nor was it tangible without any solid, and it was not solid without earth; so did the god, being the first of all things, make the corporeal by mixing together earth and fire. It was impossible for the first and the second to be combined beautifully with a third being separate; for it was necessary that in the centre there be some connection binding both of these together. And so to bind it, he, the most beautiful, made it so that in binding it would be supremely one, which brought to completion the most beautiful mathematical proportion. For whenever the middle is raised by either third numbers or powers, so that it goes first toward the same side, and then towards the far end, and then back again, and the end point towards the middle, and the middle towards the first end, and the middle and the beginning and the end becoming such that the end point and the beginning both go towards the middle, so it follows that on all sides out of necessity, that each will become one with all others. So if a plane, not having any depth, must become the body of all, the same thing then rises up from the centre, such that it is bound in the same way, and its solid nature will therefore then be manifest, the solids will never be one, for two centres always fit together, and between fire and earth the god placed water and air, and towards one another in such a way that it was possible for to bring that calculation upward towards completion, with fire adjacent to air, air adjacent to water, and as with air adjacent to water, water adjacent to earth, he bound and combined the universe to be visible and tangible. And through these ways, and the number of four units, and out of such things he begat the body of the cosmos through a harmony of proportion and he held love for these things, such that they would be bound towards him alone, inseparable by one another except to be bound by him.[14]

14. Plato, *Timaeus*, 31b4-32c4. σωματοειδὲς δὲ δὴ καὶ ὁρατὸν ἁπτόν τε δεῖ τὸ γενόμενον εἶναι, χωρισθὲν δὲ πυρὸς οὐδὲν ἄν ποτε ὁρατὸν γένοιτο, οὐδὲ ἁπτὸν ἄνευ τινὸς στερεοῦ, στερεὸν δὲ οὐκ ἄνευ γῆς: ὅθεν ἐκ πυρὸς καὶ γῆς τὸ τοῦ παντὸς ἀρχόμενος συνιστάναι σῶμα ὁ θεὸς ἐποίει. δύο δὲ μόνω καλῶς συνίστασθαι τρίτου χωρὶς οὐ δυνατόν: δεσμὸν γὰρ ἐν μέσῳ δεῖ τινα ἀμφοῖν συναγωγὸν γίγνεσθαι. δεσμῶν δὲ κάλλιστος ὃς ἂν αὑτὸν καὶ τὰ συνδούμενα ὅτι μάλιστα ἓν ποιῇ, τοῦτο δὲ πέφυκεν ἀναλογία κάλλιστα ἀποτελεῖν. ὁπόταν γὰρ ἀριθμῶν τριῶν εἴτε ὄγκων εἴτε δυνάμεων ὡντινωνοῦν ᾖ τὸ μέσον, ὅτιπερ

This system of dimensional principles described in the *Timaeus* suggests the influence of the doctrine of number principles attributed to the Pythagoreans, according to which, as explained by Tubbs (2009), the first four numbers were each representative of one of the four dimensional levels. 1 was symbolic of the point, 2 of the line, 3 of the plane, and 4 of the solid, with 10 being associated with perfection and divinity on the basis that, as the sum of 1+2+3+4, it constituted the totality of these dimensional number patterns.[15] The development of the elemental-dimensional system described by Plato from the geometric number theory of the Pythagoreans is suggested by Mugler (1948), who states that the Demiurge's plan is "inspired by the ancient dream of the Pythagoreans to explain the universe according to number…"[16] It is worthy of note that, although the fourth number belongs to what is seen to be the most complete dimensional level, the Pythagorean system still recognizes 4 as falling short of perfection. The connection of 10 with perfection and divinity may be a function of the tenth number being interpreted as an image of the intelligible simultaneity of all dimensional principles, as opposed to 4, which suggests the culmination of a sequential process of dimensional construction on the tangible level. From simultaneous completeness and perfection of all dimensional levels as defined in the intelligible, we can easily extrapolate, according to the ἁπλός unity of the cosmic model, that the dimensional schematics must also account for all mathematical laws, whether they constitute the universal

τὸ πρῶτον πρὸς αὐτό, τοῦτο αὐτὸ πρὸς τὸ ἔσχατον, καὶ πάλιν αὖθις, ὅτι τὸ ἔσχατον πρὸς τὸ μέσον, τὸ μέσον πρὸς τὸ πρῶτον, τότε τὸ μέσον μὲν πρῶτον καὶ ἔσχατον γιγνόμενον, τὸ δ᾽ ἔσχατον καὶ τὸ πρῶτον αὖ μέσα ἀμφότερα, πάνθ᾽ οὕτως ἐξ ἀνάγκης τὰ αὐτὰ εἶναι συμβήσεται, τὰ αὐτὰ δὲ γενόμενα ἀλλήλοις ἓν πάντα ἔσται. εἰ μὲν οὖν ἐπίπεδον μέν, βάθος δὲ μηδὲν ἔχον ἔδει γίγνεσθαι τὸ τοῦ παντὸς σῶμα, μία μεσότης ἂν ἐξήρκει τά τε μεθ᾽ αὑτῆς συνδεῖν καὶ ἑαυτήν, νῦν δὲ στερεοειδῆ γὰρ αὐτὸν προσῆκεν εἶναι, τὰ δὲ στερεὰ μία μὲν οὐδέποτε, δύο δὲ ἀεὶ μεσότητες συναρμόττουσιν: οὕτω δὴ πυρός τε καὶ γῆς ὕδωρ ἀέρα τε ὁ θεὸς ἐν μέσῳ θείς, καὶ πρὸς ἄλληλα καθ᾽ ὅσον ἦν δυνατὸν ἀνὰ τὸν αὐτὸν λόγον ἀπεργασάμενος, ὅτιπερ πῦρ πρὸς ἀέρα, τοῦτο ἀέρα πρὸς ὕδωρ, καὶ ὅτι ἀὴρ πρὸς ὕδωρ, ὕδωρ πρὸς γῆν, συνέδησεν καὶ συνεστήσατο οὐρανὸν ὁρατὸν καὶ ἁπτόν. καὶ διὰ ταῦτα ἔκ τε δὴ τούτων τοιούτων καὶ τὸν ἀριθμὸν τεττάρων τὸ τοῦ κόσμου σῶμα ἐγεννήθη δι᾽ ἀναλογίας ὁμολογῆσαν, φιλίαν τε ἔσχεν ἐκ τούτων, ὥστε εἰς ταὐτὸν αὑτῷ συνελθὸν ἄλυτον ὑπό του ἄλλου πλὴν ὑπὸ τοῦ συνδήσαντος γενέσθαι.

15. R. Tubbs, *What is a Number?* (Baltimore: The John Hopkins University Press, 2009), 12.

16. C. Mugler, *Platon et la Recherche Mathématique de son Époque* (Strasbourg and Zurich: *Éditions* P. H. Heitz, 1948), 82. "…dans ce plan inspiré par le vieux rêve des Pythagoriciens d'expliquer l'univers par le nombre…"

foundations of trigonometric ratios, or instead preside specifically over the behaviour of moisture in Earth's atmosphere; and in addition to the operation of numbers as dimensional patterns pertaining directly to structure and motion at the tangible level, this system must also include instances in which numbers that represent a geometric relation (such as a trigonometric ratio) do not signify spatiotemporal magnitudes, but rather instances in which a mathematical function equivalent to such a magnitude is otherwise instrumental in governing a certain aspect of the cosmos.

Given the ontological simultaneity of all dimension levels, it stands to reason that the ἁπλός schematic must immediately account for all laws and formulae pertaining to geometric angles; it seems that it must therefore also account for all principles pertaining to the mathematical definition of the circle, for Aristotle states in *Metaphysics* H that the circle is σχῆμα ἐπίπεδον.[17] One possible interpretation of this term is that of a "foundational figure," since it is discussed as a simile to Aristotle's characterization of matter as the essential aspect of objects perceptible through the senses. This interpretation also seems to be supported by Aristotle's remark in *Metaphysics* Z stating that unlike the definition of the syllable, wherein all of its elements are specified, the definition of the circle does not include the partitions of the circle.[18] The term σχῆμα ἐπίπεδον might also refer to a planar figure, which would indeed be compatible with a two-dimensional circle. Apostle states that although the circle is often defined according to the straight line, that is, by the equidistance of all lines emanating from the centre of the circle to the circumference, it is more correct to understand the circle and the straight line as being "simultaneous by nature."[19] This statement may be interpreted as indicating that principles of structure and proportion governing the circle and those governing the straight line are interdependent. Indeed, since we have demonstrated that the operations of the line and the plane ought to be similarly simultaneous, it seems to follow that the principles governing angles as defined in the ratios and divisions of the circle are necessary for the full functionality of the dimensional operations, just as the

17. Aristotle, *Metaphysics*, VIII. 6. 1045a35.

18. Aristotle, *Metaphysics*, VII. 9. 1034b. τοῦ μὲν γὰρ κύκλου ὁ λόγος οὐκ ἔχει τὸν τῶν τμημάτων, ὁ δὲ τῆς συλλαβῆς ἔχει τὸν τῶν στοιχείων.

19. H. G. Apostle, *Aristotle's Philosophy of Mathematics* (Chicago: The University of Chicago Press, 1952), 116. Op. cit. Aristotle. 92b19-22, 1407b26-28, 14b33-15a1, 142b7- 10.

dimensional principles are necessary for the definition of the circle.

Chapter 3: Ὕλη

The association of dimensional principles with the tangible elements may, furthermore, be regarded as an elaboration upon the Pythagorean doctrine of dimensional numbers, as it suggests a system of geometric foundations that accounts not only for structure, but for movement as well. As we shall observe shortly, the elements are differentiated from one another based upon the extent of their mobility, as well as their effect on the structure and motion of the other elements. Fire, for instance, which is stated to be possessed of the greatest mobility of all the elements, and to be most capable of affecting the structure of the other elements, is associated with the geometric point principle. This relation may be, at least to some extent, a function of the unconnected geometric point being unbound by any motion on the part of any other point. Earth, by contrast, is stated to be the most stable of the elements, possessed of the least degree of mobility and mutability, and is therefore associated, appropriately, with the solid principle, which includes at least four interconnected points. Each of these points, when compelled toward movement, forces the others in the structure to shift their position as well, and is similarly at the mercy of the others when they are induced toward motion. As Plato states in the passage above,[20] and as Cornford (1965) explains in his translation and commentary on the *Timaeus*,[21] fire is necessary for the entities of the sensible cosmos to be perceptible, while earth is necessary for them to be tangible. Why then must they be mediated by air and water? One possible explanation is that tangible objects require the linear dimensional principle so that they are not completely uncontrolled in their movement, and the planar dimensional principle in order to ensure that their solidity does not render them entirely immobile. The former, as we would expect, would be associated with ordered linear motion, whereby an object travels on a calculated course. The latter suggests the type of interconnected motion associated with liquids, characterized by waves, vibrations, and ripples among geometric points, as well as the displacement of liquid by an object possessed of greater hardness and density. This dimension system

20. Plato, *Timaeus*, 31b4-32c4.

21. F. MacDonald Cornford, ed. and trans. *Plato's Cosmology: The Timaeus of Plato*, The Library of Liberal Arts (Indianapolis and New York: The Bobbs-Merrill Company, Inc., 1965), 43.

thus represents a development in which the Pythagorean doctrine of dimensional numbers is articulated with greater clarity in terms of its relation to the structure and activity of sensible entities.

In order to grasp the mathematical communication of Form to tangible beings according to Plato's ontology, we must examine his theory concerning the geometric structure of matter as presented in the *Timaeus*. It stands to reason that in order for sensible entities to be defined in a complete manner at the level of the intelligible, the schematics to which they adhere must account for the magnitude and motion of their components at all levels of their construction. The question of matter is particularly significant in examining the manner in which the activity of mathematical formulae on the level of sensible beings is capable of deviating from the teleological specifications defined within the intelligible. The nature of this deviation is presented in the account of the cosmogony as set forth in the *Timaeus*, wherein the Demiurge is stated on several occasions to construct the cosmos ἐξ ἀνάγκης, or "from necessity."[22] Vlastos (1941) describes necessity, or ἀνάγκη, as "the "secondary" cause, which is "necessary," irrational, fortuitous, and disorderly."[23] As an irrational force, necessity must, in some regard, be at odds with the rational governance of the cosmic model, yet in a different sense, it must collaborate with reason. Timaeus indicates, for instance, that even after the application of precise geometric structure to matter, the difference in movement speed on the part of the elemental solids based on size, with the solids possessing the smallest sides being the most agile, is a function of necessity.[24] Since necessity continues to hold sway over matter even after its ordering, it is clear that the Demiurge has not removed necessity from the cosmos; rather, it would seem that matter has been patterned so as to take advantage of the natural inclinations of necessity for the purpose of directing matter to conform to intelligible paradigmatic specifications. From this relation between reason and necessity, it would follow that the paradigms that govern the sensible cosmos must account for the operations of necessity, and that the role of matter in the construction of the universe is defined within the intelligible.

In seeking any proper definition of matter, the greatest difficulty

22. Plato, *Timaeus*, 28a9, 32a5, 37c2.

23. G. Vlastos, "Morals, Politics, Metaphysics," in *Platonic Studies*, ed. G. Vlastos (Princeton, New Jersey: Princeton University Press, 1973), 155. Op. cit. Plato, 48a, 56c, 68e, 47e, Cf. Plato, *Philebus*, 26d6, 7.

24. Plato, *Timaeus*, 56a6-7. ταῦτ᾽ οὖν δὴ πάντα, τὸ μὲν ἔχον ὀλιγίστας βάσεις εὐκινητότατον ἀνάγκη πεφυκέναι.

with which we must contend is the characteristic of movement and flux attributed to matter in Plato's ontology. In particular, the conflict that we seek to address is between the impermanence associated with matter, and the geometric properties ascribed to them by Plato. According to the description of matter in the *Timaeus* as interpreted by Gill (1986), it is more accurate to characterize the elements not as "this," but rather as "what is such."[25] This description suggests that the elemental properties of matter are not to be understood as non-predicated objects, but rather as states that may be ascribed to tangible matter which would be otherwise indescribable. It seems then to follow that this definition should belong to the geometric patterns of the elements. The foundations of these elemental solids, as Timaeus explains, are isosceles and scalene right triangles. For the solids corresponding to fire, the Demiurge is stated to join eight scalene right triangles into four equilateral triangles, which he then fashions into a tetrahedral shape. For the air solids, an octahedron is constructed using eight equilateral triangles of the same type as those used in producing the fire solids, while twenty of these equilateral triangles are assembled into the icosahedral water solids. The earth solids are built of six square planes, each of which is a connection of four isosceles triangles.

Based on this impermanence, it is immediately evident to us that we would be incorrect to interpret the elemental solids as stable particles, and that we rather ought to understand them as patterns of movement imposed upon an otherwise indescribable mass. That is to say, the definition of the elemental solids addresses the question not of what matter is, but rather of what matter does. According to the Matrix hypothesis examined by Ostenfeld (1982), these solids are understood as belonging to a "universal Matrix." Ostenfeld presents the Matrix hypothesis as a possible interpretation of the Receptacle concept, inasmuch as the elemental solids are imprinted into the Matrix, within which their size and shape translate into properties of weight and speed. Ostenfeld expresses this type of receptacle as something of a mould, and describes it as a Matrix according to Plato's reference to the ἐκμαγεῖον, which is stated to be "shaped by the things pressed into it."[26] The elemental solids, as

25. M. L. Gill, "Matter and Flux in Plato's *Timaeus*," *Phronesis* 34 (October 1986), 34-35. Op. cit. Plato, *Timaeus*, 49c7-50a4.

26. E. N. Ostenfeld, *Form, Matter, and Mind*, Martinus Nijhoff Philosophy Library Volume 10 (The Hague: Martinus Nijhoff Publishers, 1982), 125. Op. cit. Plato, *Timaeus*, 50c2-6. In the text of the *Timaeus*, the use of the expression διασχηματιζόμενον ὑπὸ τῶν εἰσιόντων carries the implication of shapes produced as impressions within

well as the tangible entities constructed from them, would therefore constitute patterns imprinted within an otherwise indistinct mass. Ostenfeld identifies the elemental solids as atoms, and states that their tangible properties are partly a function of their belonging to the Matrix. He explains also that the primary characteristics of cosmic matter (i.e. ordered matter) are those of shape, size, and location, with attributes such as weight, motion, sharpness, and hardness being derived. Ostenfeld thereby determines that the elemental solids are not to be understood as fully geometric or corporeal, but rather as intermediate with respect to these classes. He describes the universal Matrix, furthermore, as being full on the basis that Plato refutes the presence of void within the universe described in the *Timaeus*. Ostenfeld specifies also that the solids alone do not constitute atoms, but are only understood as such in conjunction with the Matrix, such that they might be recognized. as imprints upon the Matrix. He also significantly identifies the geometric characteristics as kinematic, lending credence to the identification of the elemental solids as patterns of motion.[27]

One possible explanation for the patterns of motion represented by the elemental solids is that each of the right triangles comprising the solids constitutes a triple tuple which consists of a value equivalent to each of the x, y, and z axes, such that these are

some sort of medium, and therefore seems to suggest that Ostenfeld's universal Matrix theory is congruent with Plato's explanation of matter as a receptacle of Form. Although Ostenfeld does not explicitly specify the relation of the universal Matrix to the cosmic model, it stands to reason that, given the eternity of the cosmic model, the Matrix cannot be understood to have prior existence to the cosmic model. Since we have determined (4) that the model can never be in an incomplete state, and, as we shall later observe, it belongs essentially to the cosmic model to be instantiated at the sensible level, it also stands to reason that the instantiation of the cosmic model must be immediate, for the cosmic model would otherwise be incomplete. It then follows that the universal Matrix must immediately have instantiations of Form imprinted within it.

27. Ostenfeld, *Form, Matter, and Mind*, 125-7. Concerning the sharpness of the atoms, Ostenfeld refers to Plato, *Timaeus*, 61d-e. In this passage, Timaeus explains that the apparent heat of fire atoms is due to their sharpness, a characterization which might be seen to suggest that the atoms are in fact corporeal. The term used to imply sharpness, ὀξύ, may also be understood to signify swiftness, which in this case may be appropriate to suggest faster motion on the part of fire atoms in contrast to surrounding atoms which moveat a slower speed. Ostenfeld references Plato's refutation of void at Plato, *Timaeus*, 52e and 58a. At the former of these two passages, it is suggested that the presence of space between the elemental layers would prevent their movement. In this passage it is also indicated that the elements should not be equidistant from one another, nor should they be balanced in stasis, for they would consequently be incapable of balanced movement.

variables corresponding to a location in three-dimensional space, and are predicated of the most basic unit of matter (about which no determinate statement may be made, except for its role as that which is moved according to cosmic patterns), henceforth termed as a prime particle. Each solid would signify a relation of these prime particles, with each relation consisting of a number of prime particles equal to the number of right triangles comprising the solid, such that a fire solid is comprised of 8 prime particles, with 16 for an air solid, 40 for a water solid, and 48 for an earth solid. The solids would be unable to function properly if the variables corresponded to the edges of the triangles, as their values would then remain constant, and they would therefore be incapable of movement. The variables must then correspond to the vertices of the right triangle, such that each vertex shared by two triangles constitutes an instance of a spatial variable that is equal for two different prime particles; the edges of the triangles, meanwhile, would signify the proportion of values to one another. The distances, furthermore, might reasonably be measured according to the size of a prime particle, if they are assumed to be of the same size. Throughout the movement of the prime particles, the variable proportions would remain, as would the equalities of the spatial variables between particles. The disintegration and reintegration of particles might be explained by possible disruptions in proportion based on proximity to other solids, since absolute void is absent from the cosmos described in the *Timaeus*. There is, however, little evidence for this theory apart from its apparent functional plausibility, as the *Timaeus* provides no conclusive indication of its correctness, beyond its possible accuracy according to the negative statements which may be made about matter based on identity as a state rather than an entity. In this theory, we assume that each of the elemental solids represents a mathematically determined pattern governing prime particles, yet within Plato's account, there is no indication of tangible units of this type.

Nevertheless, given that the elemental solids appear to be correctly identified as patterns of motion rather than as stable entities, it would seem there is a "minimum tangible unit," or MTU, of some sort which is governed by these patterns. Ostenfeld does not suggest the presence of such units within his universal Matrix, yet the absence of void within this system implies that the elemental solids are capable of moving through one another. If the universal Matrix signifies a single undivided mass that is

not comprised of minimum tangible units, and each elemental solid represents a different pattern of motion within the Matrix, then we will find it to be rather difficult, if not entirely impossible to explain the delineation of different motion patterns within the Matrix. Since Plato indicates that no definitive specification may be made regarding the indistinct mass of pre-cosmic matter, it stands to reason that any distinction of minimum tangible units must be assumed to have occurred in the ordering of matter into its cosmic state by the Demiurge. Indeed, as we have observed previously, the first delineation of matter seems to have occurred with the initial separation of the elements as described earlier in the *Timaeus*.[28] Given that this delineation constitutes the application of precise dimensional properties to the pre-cosmic mass, it stands to reason that the elemental separation of matter does not, as we have stated before, imply the separation of pre-cosmic matter into four discrete sections, but rather the exact distinction of all possible patterns of movement within matter. Based on the fact that the elemental solids must, by definition, account for all four of the initial dimensional levels, each type of solid must adhere to the dimensional paradigms of all of the elements. Given, however the association of each element with one of the dimensional levels, it would seem that patterns of motion associated with cosmic matter are categorized in terms of the number of minimum tangible units moving together within each type of pattern. As such, the tetrahedral fire solids represent the smallest possible measure of interdependence among MTUs. Even the pyromorphic elemental state requires the calculated spatial relation of small numbers of MTUs to one another, for without this connection, they may be unable to function in concert in the manner required for the construction of tangible entities according to the schematics of the cosmic model. Nevertheless, the structure of the fire solid is such that compared with the other types of elemental solids, each MTU comprising the structure of the solid has a smaller burden in terms of the number of other such particles that will be bound to its path of spatial motion. The fire solid may thus be said, out of all elemental solids, to adhere to the dimensional paradigm of the point in the most direct manner, as it contains all basic dimensional levels, yet is fettered to the least possible extent in terms of interdependencies. The octahedral air solid, possessing twice the number of interdependencies as that of the fire solid,

28. Plato, *Timaeus*, 31b4-32c4.

is reminiscent of the linear connection of the second dimensional level. The icosahedral water solid, as with the plane, suggests a manner of movement analogous to the waves that characterize the behaviour of liquid, being similarly prone to displacement by sharper, harder, and more agile solids such as those of fire.

In our consideration of the elemental solids as patterns of motion for the minimal tangible units, we cannot ignore the question of infinite divisibility on the part of both the former and the latter; for the presence or absence of this property is of critical importance in explaining the behaviour of matter according to Plato, and remains an area of ambiguity in the interpretation of Plato's ontology. Vlastos (1975) appears to reject the need for the elemental solids to be infinitely divisible, stating that "So far as logic goes, there is no warrant for the reasoning, '*X* is made up of a million *Y*'s; *X* is divisible; *ergo* each of the *Y*'s is divisible.'"[29] The result, however, of the elemental solids or the minimum tangible units not being infinitely divisible, proves to be mathematically untenable; as Hett (1936) indicates in the commentary accompanying Aristotle's discussion of indivisible lines, the absence of midpoints on the linear pathways of the solids would dictate that any traversal of the pathways would have to occur without passing through any intermediate point, a manner of movement which would be impossible.[30] Thus, in order for each MTU to be capable of traversing spatial distance measurable in fractions of the particles' own size, it must be infinitely divisible, and each of the elemental solids must be infinitely divisible inasmuch as each x, y, or z value represented in each of the vertices may be divided into infinitesimally smaller factions of itself; for the MTUs belonging to the solid would otherwise be incapable of motion. Thus, the MTUs are not indivisible in the mathematical sense, but rather in the ontological sense, insomuch as any division of a minimum tangible unit would be an incomplete object, as it would be defined by that of which it is a partition.

By this point it will be evident to us that while the division of matter and the construction of the elemental solids may be implied as a contemporaneous occurrence, the transformation that occurs is describable by two aspects which are inextricably bound to one another. The first aspect, described earlier in the

29. G. Vlastos, *Plato's Universe* (Seattle: University of Washington Press, 1975), 68-9.

30. W. S. Hett, ed. and trans, "On Indivisible Lines," in *Aristotle Minor Works*, Loeb Classical Library 307 (Cambridge, Massachusetts and London, England: Harvard University Press, 1936), 426.

Timeaus, is that of the imposition of finite magnitude and three-dimensional structure upon pre-cosmic matter as determined by the need for created entities to be both perceptible and tangible. The second aspect is that of mathematically calculated, cooperative motion, whereby the activity of minimum tangible units is coordinated at the most minute level possible, so as to function with the singularity of purpose required for the coalescence of matter into the stable systems of sensible being. While the former aspect is required at the tangible level in order for the latter to be possible, both such aspects must, in accordance with the ἁπλός unity of the cosmic schematic, be both simultaneously defined and non-sequential in terms of their relation to one another. This leads us to postulate that the four elemental principles may be most accurately described as intricate algorithms governing the structure and motion of matter, such that each element represents a different theme and subordinate objective, and yet, no element can carry out its objective without the others. Given the crucial role of all of the elements in the construction of three-dimension shapes, it may be said that each of the elemental solids requires the collaboration of all four elements, both for their involvement in the determination of its dimensional characteristics, as well as the need for the interaction of all types of elemental solids in bringing the tangible realm into adherence with the cosmic model.

From this observation it is readily evident that the elemental solids as they are described in the *Timaeus* must be defined in some respect within the model of the cosmos. Just as Form must be brought to its completion through application to matter, it has been suggested that it belongs essentially to matter to be subordinated to Form. As Kutash (2011) explains, such a position is presented by Proclus, who maintains that "matter is not simply a passive *hupokeimenon*, a recipient of Form prior to its activation, but an active component and even opponent of Form."[31] Assuming that Proclus' position is correct, we must infer that matter must indeed be defined on the level of the intelligible, and that the mathematical operability of matter requires that it allow for the possibility of conflicting with the Good. We are therefore faced with a strange paradox, for since matter must allow for all mathematical variations that are possible given the properties of its movement, it must also allow for deviations which are contrary to the schematics of the intelligible. We consider here the interplay of reason and necessity, and the

31. E. Kutash, *Ten Gifts of the Demiurge: Proclus on Plato's Timaeus* (New York: Bristol Classical Press, 2011), 44.

manner in which the former essentially mandates this interaction.

In considering the mathematical operation of matter in Plato's ontology, we must also account for the imperfections that are manifest on the level of tangible entities. Although the objects of sensation, as Aristotle explains, do not constitute perfect representations of geometric abstracts,[32] the accidental variations that occur at a sensible level, since they are confined to the spatiotemporal limitations of the corporeal realm, must still be restricted by mathematical parameters. It may, furthermore, be the case that if the elemental solids were restricted in such a way as to adhere seamlessly to the structural parameters of intelligible paradigms, they would lack the capacity to carry out the patterns of motion defined within the paradigms. Mason (2006) explains that according to Plato's ontology, objects on the level of αἴσθησις function according to necessity, which, when ungoverned by a rational pattern, acts in a disorderly manner, but which is also capable of being exploited by intellect for the purpose of cosmic structure.[33] Given the requirement, however, that sensible entities must differ in their activity in accordance with specific conditions, it seems to follow that sensible objects are necessarily capable of acting in a manner different from that which is suitable to the circumstance in question; for in order for sensible objects to act in accordance with a rational pattern, it stands to reason that their behaviour must be dictated by the parameters of specific scenarios, and that the actions of which they are capable, as well as the conditions under which these actions are to be performed, are defined on the level of paradigm. We therefore observe a curious paradox, for it seems that in order for the schematics defined within the intelligible to be brought to the full extent of their perfection, they must be imperfectly instantiated at the tangible level, and thus compelled to seek closer adherence to the paradigms from whence they originate.

Chapter 4: Conclusion

It is therefore clear to us that the mathematical ordering of matter as detailed by Plato constitutes nothing less than the communication

32. Aristotle, *Metaphysics*, II. 2. 997b15-22. ὥστ᾽ ἐπείπερ ἡ ἀστρολογία μία τούτων ἐστίν, ἔσται τις καὶ οὐρανὸς παρὰ τὸν αἰσθητὸν οὐρανὸν καὶ ἥλιός τε καὶ σελήνη καὶ τἆλλα ὁμοίως τὰ κατὰ τὸν οὐρανόν. καίτοι πῶς δεῖ πιστεῦσαι τούτοις; οὐδὲ γὰρ ἀκίνητον εὔλογον εἶναι, κινούμενον δὲ καὶ παντελῶς ἀδύνατον.

33. A. S. Mason, "Plato on Necessity and Chaos," *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* 127.2, Selected Papers from the American Philosophical Association, Pacific Division, 2004 Meeting (January 2006), 284.

of the intelligible to the tangible. This expression occurs in terms of both the delineation of spatial dimensions as well as the organization of the foundations of structure and movement for sensible beings. The identification of the elemental solids as patterns of motion on the part of minimum tangible units emphasizes the adherence of tangible entities to the schematics according to which they are constructed as an activity. Thus, it follows that every sensible being is in fact a composition of geometrically regulated composition of motions. Perhaps even more significantly, this ordering represents a type of unification, wherein several minimum tangible units are connected as participants in a single structure, while at the same time maintaining their distinction in terms of the *x*, *y*, and *z* values associated with them. This cohesion translates, in turn, into the unified structures of tangible beings. Unified pluralities of this sort, are, however, composed, representing a ποικίλα construction, in contrast to the ἁπλός unity of the cosmic model, which may itself be identified as the intelligible definition of unified plurality.

The same type of mathematical regulation as that which is carried out at the level of matter may also be interpreted as a principle of order and perfection at several other tiers of tangible being, and it is in this respect that we may identify the connection between mathematics and justice, and thereby gain insight into the dialogue between the ontological, political, and ethical dimensions of Plato's philosophy. Our initial evidence of the nature of this connection is in Plato's choice of language in describing the ordering of matter in the *Timaeus*, as he characterizes the cosmic state of matter using terminology associated with military formations,[34] which may also be a reference to the importance of mathematical knowledge in military strategy as detailed in the *Republic*.[35] Plato illustrates in the *Republic* the manner in which

34. S. Broadie, *Nature and Divinity in Plato's Timaeus* (Cambridge: Cambridge University Press,2012), 182. Op. cit. Plato, *Timaeus*, 48a5-7; cf.30a2-6. In the earlier of these passages, the Demiurge is described as having arrayed the elements into τάξις, a term which may also identified with the combat formation of soldiers (Liddell and Scott, 1996), suggesting the imposition of order upon the elements implies bringing them into a structured pattern of motion. In the later passage, Timaeus describes the elements as the "wandering cause," a term possibly intended to emphasize the connection of the principles that will ultimately govern their movement with the mathematical laws that guide the heavenly spheres, since these are also termed as "wanderers"(Plato, *Timaeus*, 38c6).

35. Plato, *Republic*, VII. 527d2-6. ἐμοὶ γοῦν, ἔφη: τὸ γὰρ περὶ ὥρας εὐαισθητοτέρως ἔχειν καὶ μηνῶν καὶ ἐνιαυτῶν οὐ μόνον γεωργίᾳ οὐδὲ ναυτιλίᾳ προσήκει, ἀλλὰ καὶ στρατηγίᾳ οὐχ ἧττον. ἡδὺς εἶ, ἦν δ᾽ ἐγώ, ὅτι ἔοικας δεδιότι τοὺς πολλούς, μὴ δοκῇς ἄχρηστα μαθήματα προστάττειν

mathematical principles are required for the correct execution of the multitude of practical tasks necessary for the livelihood of the polis through his explanation of the particular importance of the mathematical science of astronomy, which is stated thus:

> 'But why? Should we place astronomy third among these? That does not seem correct.'
>
> 'It seems so to me,' he said, 'for it is necessary for the accurate perception of hours, months, and yearly cycles, not only for farming and sailing, but also just as much so for military strategy.'[36]

Since the mathematical analysis of the motion of heavenly objects is understood to be of vital importance to necessities of agrarian, nautical, and military pursuits, it then stands to reason that skill in calculation of this sort is similarly crucial for the leaders of the political community, that they may order the livelihood of society to function in concordance with the cycles of the natural world.

The governance of society according to the laws of proportion is, furthermore, necessary in order to ensure that the polis is able to maintain the resources needed for their survival. In a warning that proves to be frighteningly prescient, Plato speaks in an earlier passage of the *Republic* about the plight of the luxurious society, which, as a consequence of its excesses, suffers from ill health, and is plagued with resource shortages that drive it into armed conflict with its neighbours. Within the luxurious society, the avarice of the public demand the expansion of the city's territory, and the citizens of such a community, either disregarding or having forgotten the essential finite boundaries articulated within the dimensional paradigms, seek an infinite abundance of spatially finite objects.[37]

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36. Plato, *Republic*, VII. 527d1-4. τί δέ; τρίτον θῶμεν ἀστρονομίαν; ἢ οὐ δοκεῖ;

37. Plato, *Republic*, II. 373d1-e7.

οὐκοῦν καὶ ἰατρῶν ἐν χρείαις ἐσόμεθα πολὺ μᾶλλον οὕτω διαιτώμενοι ἢ ὡς τὸ πρότερον;

πολύ γε.

καὶ ἡ χώρα γέ που, ἡ τότε ἱκανὴ τρέφειν τοὺς τότε, σμικρὰ δὴ ἐξ ἱκανῆς ἔσται. ἢ πῶς λέγομεν;

οὕτως, ἔφη.

οὐκοῦν τῆς τῶν πλησίον χώρας ἡμῖν ἀποτμητέον, εἰ μέλλομεν ἱκανὴν ἕξειν νέμειν τε καὶ ἀροῦν, καὶ

ἐκείνοις αὖ τῆς ἡμετέρας, ἐὰν καὶ ἐκεῖνοι ἀφῶσιν αὐτοὺς ἐπὶ χρημάτων κτῆσιν ἄπειρον, ὑπερβάντες τὸν τῶν ἀναγκαίων ὅρον;

The luxurious city thereby falls out of balance with its surroundings, for not only does it displace and destroy neighbouring communities, but it consumes natural resources more quickly than they are able to replenish themselves. The appetitive citizens of this *polis* fail to recognize that they are subject to the same laws of proportion that govern the interactions of the elemental solids and the MTUs that comprise them. Just as the movement and mutation of elemental solids contrary to the cosmic ordering of the Demiurge is disruptive to the structure and activity of larger tangible entities, so too will mathematically disproportionate behaviour on the part of these latter entities cause disturbances to the systems of which they are components. Similarly, when a political community violates the sustainable proportions of position within the natural infrastructure to which it belongs, it disrupts that which supports its existence.

However, through apprehension of the foundational dimensional principles and the heavenly orbits, mathematical knowledge informs just existence based on awareness of the finitude of tangible substance, as well as a comprehensive understanding of the cycles governing the functionality of nature. As such, the just person will be wise enough to eschew the excesses of the luxurious city,[38] whose inhabitants, through their folly, face eventual death, whether by starvation or by violent conflict with their neighbours. When a political community is governed by those with correct apprehension of the activity of nature as governed by temporal cycles,[39] and of the finitude and perishability of sensible substance, it has the capacity to be regulated in such a way that its use of natural resources does not outpace the cycles according to which they are replenished. In the *Laws*, Plato offers a precise example of the manner in which such knowledge might be applied to the task of statecraft. The city of Magnesia, as noted before, is structured so that it has a population of 50, 000, with a total of 5,040 households, with the latter of the two numbers being considered particularly suitable for the division of wealth and labour due to the fact that it has 59 divisors, including the first 10 positive integers.[40] As explained by Planinc (1991), it is through the study of geometry

πολλὴ ἀνάγκη, ἔφη, ὦ Σώκρατες.

πολεμήσομεν δὴ τὸ μετὰ τοῦτο, ὦ Γλαύκων; ἢ πῶς ἔσται;

οὕτως, ἔφη.

38. *Ibid.*

39. Plato. *Timaeus*, 47a6-7. χρόνου δὲ ἔννοιαν περί τε τῆς τοῦ παντὸς φύσεως ζήτησιν ἔδοσαν.

40. G. G. Szpiro, *Numbers Rule: The Vexing Mathematics of Democracy, from Plato to the Present.* (Oxford and Princeton: Princeton University Press, 2010), 5.

and astronomy that the citizens of Magnesia come to understand the manner in which the growth of their city follows as circular pattern.[41] Thus the work of the Nocturnal Council becomes akin to that of the Demiurge; for their leadership has the effect of raising the citizens of Magnesia towards the perfection of their existence as intelligibly defined, while ordering them in a manner that might be likened to the Demiurge's ordering of matter towards a cosmic state.

It therefore seems most accurate to describe the image of justice in Plato's thought as a system of proportions. In keeping with the rules that we have previously identified concerning the structure of schematics within the cosmic model, particularly that which precludes sequential connections, it stands to reason that this system does not constitute a linear or hierarchical structure, but rather a circular principle of relation, such that all distinct principles contained therein share the same interdependence with respect to one another. Just as this structure regulates several principles internally within itself, it must also reach outward to the realm of the sensible, drawing tangible objects toward it as it communicates rational order to them. The identification of circular pattern as a balancing principle of just governance is discussed by Carone (2005), who observes within the *Republic* the significance of astronomy in apprehending the orbits of the heavens as models of the rational consistency whose example we ought to emulate in thought and conduct.[42] In this manner, our activities become more akin to the cyclical movement of planets as opposed to the erratic motion of the elemental solids; for these solids are inclined to deviate from the intelligible schematics governing the structures into which they are assembled, and are at odds with one another, with the faster and sharper solids dispersing the cohesion of their more ponderous neighbours. Thus, through the calculated proportional ordering of our existence, we are able function properly within the natural and political structure to which we belong, and by allowing for the indefinite survival those systems, we move toward closer emulation of the eternity of the cosmic model.

41. Z. Planinc, *Plato's Political Philosophy: Prudence in the Republic and in the Laws* (Columbia and London: University of Missouri Press, 1991), 215. Op. cit. Plato, *Laws*, V.747b, VII. 809c-e, 817e-18d, 821a-22c.

42. G. R. Corone, *Plato's Cosmology and Its Ethical Dimensions* (New York: Cambridge University Press, 2005), 77. Op. cit. Plato, *Republic*, IX. 573b ff.

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